# HOMEWORK #1

Due: October 20 (Thursday) by midnight

Submit your solutions, i.e., your brief report in the form of a *single* PDF file (no Word files will be accepted!) and your MATLAB code(s) in the form of a *single* m-file via Email to the instructor. Late submissions will not be considered.

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1. Consider a *smooth extension* $f$ of the function $f_0$ defined as $f_0(x) = \exp\left[-\frac{1}{(1-x^2)^2}\right]$ to the entire real line,

   (a) plot the function $f$ and its derivative $f'$ (computed analytically) in the neighbourhood $[x_0 - a, x_0 + a]$ of the point $x_0 = 1.0$ using, for example, $a = 0.2$,

   (b) use the *Complex Step Differentiation* approach to approximate the derivative of $f$ at the points $x_1 = 0.5$ and $x_2 = 1.0$ for different step sizes $h \in [10^{-10}, 10^{-2}]$; plot the errors, relative or absolute as appropriate, as a function of $h$; what explains the different behavior of the errors at the points $x_1$ and $x_2$?

   *[5 points]*

2. Derive a *complex-step* differentiation formula using the complex number $\Delta z = e^{i\frac{\pi}{4}}$ as the perturbation of the argument (instead of $\Delta z = i$). Note that it will have two different versions and use both of them to approximate the derivative of the function $f$ from the previous example. Plot the errors using the same values of step size $h$. Derivation of the formulas should be included in the "text" part of your submission.

   What is the order of accuracy of the obtained methods? How to explain the different behavior of the errors in the two versions?
   *[5 points]*

3. Consider the following two-point boundary-value problem

$$\begin{aligned} -\frac{d^2y(x)}{dx^2} &= g(x) \qquad x \in (0, 2\pi), \\ y(0) &= -1, \\ y(2\pi) &= 1, \end{aligned} \tag{1}$$

   with two different right-hand side functions

$$g(x) = x^3 + 2x + 1, \tag{2}$$

$$g(x) = x^4 + 2x + 1. \tag{3}$$

   Solve problems (1)–(2) and (1)–(3) numerically using (i) the second-order central difference method and (ii) the compact approach. Plot the error defined as

$$E = \|y_h - y\|_{L_\infty(0,2\pi)}$$

   as a function of the step size $h$, where $y$ denotes the analytical solution and $y_h$ is the numerical solution obtained with the step size $h$. Use the number of grid points in the range $[10, 10^4]$.

   How to explain the difference in the behavior of the error of the solutions obtained with the compact approach and corresponding to (2) and (3)?

   *[5 points]*