

QUIZ #1

2:00pm, October 24 (Monday), 20 minutes, 9 points max
(no books, no notes)

Write your name and Email address on the top of this sheet
Write your answers on the reverse side and/or attach additional sheets as
necessary.

1. You are given the following initial-value problem

$$\frac{d^2y}{dt^2} = -y, \quad y(0) = 1, \quad \frac{dy}{dt}(0) = 0.$$

Discretize it using

- (a) Euler's explicit method,
- (b) leapfrog method.

Present the solutions using matrix notation.

(3 points)

2. You are given a (stable) system of ODEs $\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x}$, where $\mathbf{x} \in \mathbb{R}^N$, $N > 1$. Without performing actual calculations, state the steps required in order to determine the maximum time step Δt allowed when using an explicit Euler method to discretize this problem.

(3 points)

3. Consider an element $\mathbf{x} \in \mathbb{R}^3$ and the subspace $H_2 \subset \mathbb{R}^3$ such that $H_2 = \text{span}\{\mathbf{e}_1, \mathbf{e}_3\}$, where \mathbf{e}_1 and \mathbf{e}_3 are the unit vectors associated with the x_1 and x_3 axes of the coordinate system. Find the element $\mathbf{a} \in H_2$, defined by the relation

$$\|\mathbf{x} - \mathbf{a}\| = \inf_{\mathbf{y} \in H_2} \|\mathbf{x} - \mathbf{y}\|, \tag{1}$$

in the following three cases

(a) $\mathbf{x} = [1 \ 2 \ 3]^T$,

(b) $\mathbf{x} = [1 \ 0 \ 3]^T$,

(c) $\mathbf{x} = [0 \ 2 \ 0]^T$.

(3 points)