QUIZ #2

2:00pm, November 28 (Monday), 20 minutes, 10 points max

(no books, no notes)

Write your name and Email address on the top of this sheet

Write your answers on the reverse side and/or attach additional sheets as necessary.

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1. You are given the function u defined on a periodic domain $[0, 2\pi]$. Assuming that this domain is discretized with equispaced points $\{x_j\}_{j=1}^{M}$ for some $M > 0$, explain how to evaluate $u^2(x_j)$, $1 \leq j \leq M$, i.e., the square of this function in the physical domain, using

   (a) the spectral Galerkin approach, and

   (b) the pseudospectral approach.

   In both cases specify how the computational cost depends on the number of grid points M. Comment on the presence and possible treatment of aliasing errors.
   (6 points)

2. You are given the following partial differential equation

$$\begin{aligned} &\partial_t u + c\,\partial_x u - \nu\,\partial_x^2 u = f && \text{in } [-\pi, \pi] \times (0, T] \\ &u = u_0 && \text{in } [-\pi, \pi] \text{ at } t = 0 \end{aligned}$$

   with periodic boundary conditions and where $c, \nu > 0$ are constants and $f : [-\pi, \pi] \times [0, T] \to \mathbb{R}$. Assuming that the solution is approximated as $u(t, x) \approx u_N(t, x) = \sum_{k=-N}^{N} \hat{u}_k e^{ikx}$, write the equations satisfied by the Fourier coefficients $\hat{u}_k$, $-N \leq k \leq N$ when the problem is solved numerically with a spectral Galerkin method and a suitable time-discretization is adopted.
   (4 points)