HOMEWORK #1

Due: October 24 (Wednesday) by midnight

Submit your solutions, i.e., your brief report in the form of a *single* PDF file (no Word files will be accepted!) and your MATLAB code(s) in the form of a *single* m-file via Email to the instructor. Late submissions will not be considered.

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1. You are given the function

$$f(x) = \begin{cases} x^2, & x < 0 \\ x^4, & x \geq 0 \end{cases}. \tag{1}$$

Approximate the derivative of $f(x)$ at (a) $x = 0.0$ and (b) $x = 1.0$ using

- the first-order forward difference formula,
- the second-order forward difference formula,
- the second-order central difference formula,
- the fourth-order central difference formula, and
- the complex-step formula.

In each case the step size h should vary from 10^{-10} to 10^{-1}. Plot the errors with respect to the exact value of the derivative as functions of h using logarithmic coordinates on two separate plots corresponding to (a) and (b). Identify and explain unexpected behavior evident in these plots.
[3 points]

2. Consider the following two-point boundary-value problem from Quiz #1, i.e.,

$$y - \frac{d^2y}{dx^2} = g, \quad \text{in } (-1, 1), \tag{2a}$$

$$y = a, \quad \text{for } x = -1, \tag{2b}$$

$$y + \frac{dy}{dx} = b, \quad \text{for } x = 1. \tag{2c}$$

Solve this problem numerically assuming that $g(x) = x^2$, $a = 2$ and $b = 1$. Use the second-order central difference formula to approximate the equation (2a) and a first-order difference formula for the boundary condition (2c). Use the number of grid points in the range $[10, 10^4]$ and plot the error defined as

$$E(h) = \|y_h - y\|_{L^\infty(-1,1)},$$

where y denotes the exact analytical solution and y_h is the numerical solution obtained with the step size h, as a function of the step size h. In a separate figure plot the difference between the numerical and exact solutions, y_h and y, as a function of the coordinate x for all considered values of the step size h.
[5 points]

3. You are given the ODE
$$\frac{dy}{dt} = -y^3, \qquad y(0) = 10. \tag{3}$$
Determine approximately the largest step size h which can be used to integrate system (3) using the RK4 method. This step size should be determined with the accuracy of $\Delta h = 5 \cdot 10^{-4}$ by performing suitable numerical experiments. Consider step sizes $h > 0.01$ and plot numerical solutions obtained with different h as functions of time t together with the exact solution.
[4 points]