

QUIZ #1

9:30am, October 17 (Wednesday), 20 minutes, 10 points max
(no books, no notes)

Write your name and Email address on the top of this sheet
Write your answers on the reverse side and/or attach additional sheets as
necessary.

1. You are given the following boundary-value problem

$$y - \frac{d^2y}{dx^2} = g, \quad \text{in } (-1, 1), \quad (1a)$$

$$y = a, \quad \text{for } x = -1, \quad (1b)$$

$$y + \frac{dy}{dx} = b, \quad \text{for } x = 1, \quad (1c)$$

where $g : (-1, 1) \rightarrow \mathbb{R}$ is a given function and $a, b \in \mathbb{R}$ are given constants. Assuming that the domain $(-1, 1)$ is discretized with four equispaced grid points $-1 < x_1 < x_2 < x_3 < x_4 < 1$, obtain the algebraic system representing a discretization of problem (1). Use suitable second-order and first-order finite-difference formulas to approximate the differential operators in the equation and the boundary condition respectively. Is the matrix in this system singular?

(6 points)

2. You are given a system of ODEs

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (2)$$

What is the largest step size h you can use to solve this problem numerically when it is discretized with

- (a) Euler's explicit scheme, and
- (b) the leapfrog scheme?

Justify your answer.

(4 points)