MATH 749 — MATHEMATICAL AND COMPUTATIONAL FLUID DYNAMICS

Time & Place — Wednesdays 10:30–12:30 and Fridays 12:30–13:30 in HH/410

Instructor: Dr. Bartosz Protas
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Course Webpage: http://www.math.mcmaster.ca/~bprotas/MATH749

Outline of the Course: In this course we will survey mathematical and computational aspects of incompressible fluid mechanics. The course will focus on the development and properties of mathematical models of fluid flows, such as the Euler and Navier-Stokes equations, and its various simplifications relevant to potential, creeping and boundary-layer flows. In addition to presenting standard theories and known results we will also discuss a number of open problems. In the second part of the course we will review computational approaches relevant to the study of fluid flows emphasizing challenges specific to this field.

The specific topics that will be discussed include (optimistic variant):

1. Conservation of Mass and Momentum
   (a) Eulerian and Lagrangian Descriptions
   (b) Euler and Navier-Stokes Equations
   (c) boundary and initial conditions
2. Vortex Motion
   (a) vorticity and circulation
   (b) Helmholtz' Laws
   (c) N-vortex problem
3. Approximations
   (a) potential flows
   (b) Stokes flows
   (c) boundary layers
4. Computational Fluid Dynamics
   (a) discretization techniques for PDEs
   (b) enforcing incompressibility
   (c) vortex methods

Primary Reference:

1. S. Childress, "An Introduction to Theoretical Fluid Mechanics", American Mathematical Society (Courant Lecture Notes in Mathematics), 2009 (ISBN 978-0821848883).

Supplemental Reference:

2. D. J. Acheson, "Elementary Fluid Dynamics", Oxford University Press, 2009, (ISBN 0198596790).
3. P. Wesseling, "Principles of Computational Fluid Dynamics", Springer, 2001 (ISBN 3540678530).
4. J. Serrin, "Mathematical Principles of Classical Fluid Mechanics" in *Encyclopedia of Physics / Handbuch der Physik* Volume 3 / 8 / 1, 1959, pp 125-263.

In addition to the above references, sets of lecture notes and example MATLAB codes will be made available to students on the course webpage.

Prerequisites: Partial Differential Equations, elementary Physics, Numerical Analysis with basic programming skills in MATLAB

Grades: The final grades will be based on:

- two 20 min quizzes ($2 \times 10\% = 20\%$),
- two homework assignments ($2 \times 15\% = 30\%$),
- a take–home final project (50%).

The *tentative* quiz and homework due dates:

i) Quiz #1 — Friday, October 25
ii) Quiz #2 — Friday, November 29
iii) Homework Assignment #1 — Wednesday, October 2 (distributed) $\Longrightarrow$ Wednesday, October 9 (due)
iv) Homework Assignment #2 — Wednesday, October 30 (distributed) $\Longrightarrow$ Wednesday, November 6 (due)

I reserve the right to alter your final grade, in which case, however, the grade may only be increased.

Senate Policy Statement: The course is regulated by the following documents: *Statement on Academic Ethics* and *Senate Resolutions on Academic Dishonesty.* Any student who infringes one of these resolutions will be treated according to the published policy. In particular, academic dishonesty includes: (1) plagiarism, e.g. the submission of work that is not one's own, (2) improper collaboration in group work on home assignments, (3) copying or using unauthorized aids tests and examinations. It is your responsibility to understand what constitutes academic dishonesty, referring to *Academic Integrity Policy.*

Important Notice: The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.