MATH 749 - QUIZ #2 - SOLUTIONS

(1) Statement of the initial-boundary value problem for

(a) Euler equation

$$\left.\begin{aligned} &\frac{\partial \bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = -\frac{1}{\rho}\nabla p \\ &\nabla\cdot\bar{u} = 0 \end{aligned}\right\} \text{ on } \underline{\Omega}\times(t,T]$$

$\bar{n}\cdot\bar{u} = 0$ on $\partial\underline{\Omega}\times(t,T]$

$\bar{n}$ - normal vector

$\bar{u} = \bar{u}_0$ on $\underline{\Omega}$ at $t=0$

$\nabla\cdot\bar{u}_0 = 0$

(b) Navier - Stokes system

$$\left.\begin{aligned} &\frac{\partial \bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = -\frac{1}{\rho}\nabla p + \nu\Delta\bar{u} \\ &\nabla\cdot\bar{u} = 0 \end{aligned}\right\} \text{ on } \underline{\Omega}\times(0,T]$$

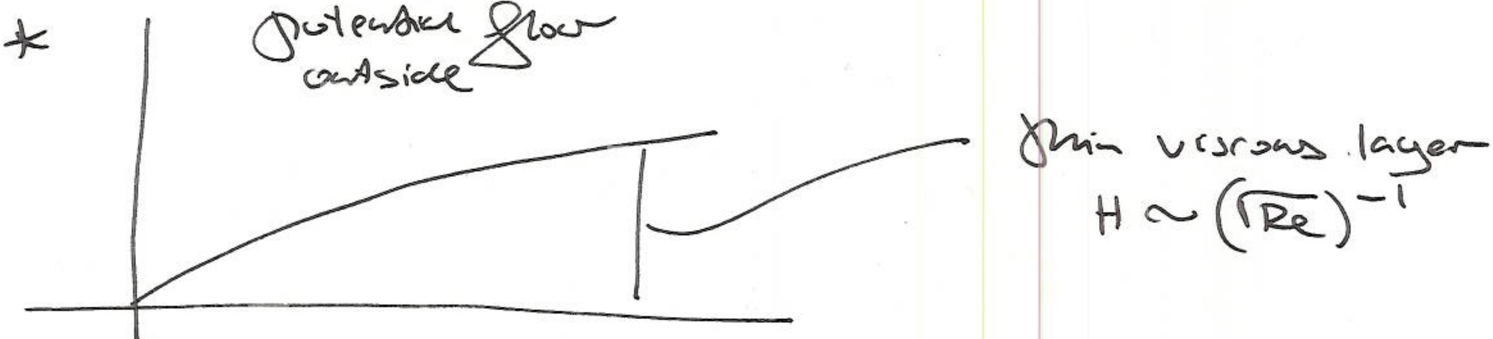
$\bar{u} = 0$ on $\partial\underline{\Omega}\times(0,T]$

$\bar{u} = \bar{u}_0$ on $\underline{\Omega}$ at $t=0$

(2a) Assumptions for the potential flow with $\bar{u}=\nabla\phi$

- incompressibility: $\nabla\cdot\bar{u}=0 \Rightarrow \Delta\phi=0$
- irrotational: $\nabla\times\bar{u}=0 \Rightarrow \bar{u}=\nabla\phi$

(2b) Assumptions for Prandtl boundary layer equations

* incompressible flow
* $Re\to\infty$ $(\nu\to 0)$
* 

* flow time-independent and attached