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ABSTRACTS
(in alphabetic order by speaker surname)

Convergence of approximation schemes for weakly nonlocal second order equations

Parsiad Azimzadeh (University of Waterloo) and Erhan Bayraktar (University of Michigan)*

A large number of interesting financial phenomena, continuous time games, motion by mean curvature, front propagation of surfaces, etc. are captured by nonlinear second order elliptic partial differential equations (PDEs). Such PDEs do not generally admit smooth solutions. As such, we look for other meaningful solutions, such as (nonsmooth) viscosity solutions. In order to compute viscosity solutions (e.g., via finite difference, volume, or element methods) one can invoke the seminal result that a monotone, stable, and consistent numerical scheme converges to the viscosity solution if the limiting PDE satisfies a maximum (a.k.a. comparison) principle. However, this result is established only for local PDEs and can be applied only to certain special nonlocal PDEs. In this talk, we extend this result to weakly nonlocal PDEs in a very general manner and give a financially motivated example from optimal stochastic impulse control.

A conservative finite volume scheme for Poisson-Nernst-Planck equations on adaptive moving mesh grids

Xiulei Cao (York)

We deal with the numerical investigation of Poisson-Nernst-Planck (PNP) equations which describe the dynamics of ion transport in ion channels. We present a finite volume discretization for solving the PNP equations. To decrease the computational cost, an adaptive moving mesh method is employed. We propose two monitor functions in the moving mesh partial differential equations to increase the accuracy of the implementation. This discretization scheme guarantees positivity property of the numerical solution and keeps the concentration of ions conservative. In the final part, numerical experiments are conducted to confirm the theoretical analysis.

Multigrid methods for the Monge-Ampère equations

Yangang Chen (Waterloo)

Monge-Ampre Equations arise from mass transport, image processing, mesh generation, etc. Multigrid methods are efficient fast solvers for elliptic PDEs. We propose multigrid methods for convergent mixed finite difference discretization for two dimensional Monge-Ampre equation. Numerical experiments show that the convergence rates of the proposed multigrid methods are mesh-independent.

Valuing of two-asset option: comparison between numerical PDE and analytical approach
Yuwei Chen (Toronto)
We consider the pricing of two-asset options by numerical Partial Differential Equation (PDE) methods, and compare the results with certain analytical formulae. Two cases of options are discussed: exchange option and spread option. For exchange options, the analytical formula considered is the (exact) Margrabe formula. For spread options, we considered the approximated analytical solutions, Kirk's formula and the formula by Li and Deng. For both of exchange option and spread options, the basic numerical PDE model is the two-dimensional Black-Scholes PDE. Different types of boundary conditions are studied and compared to analytical formulae. The effect of the boundary conditions on the solution at various points of the grid is studied. Furthermore, various types of non-uniform grids are considered, aiming at reducing the error at certain areas of the grid. The experiments indicate that the numerical PDE computed price and Greeks are second-order, for appropriately chosen grid discretization. We also discuss the effect of certain problem parameters to the accuracy and convergence of the solutions.

Numerical functional integration on GPUs: how we may actually calculate the incalculable!
Nike Dattani (McMaster)
Feynman's path integral is equivalent to the Schrdinger equation, which is the poster equation for quantum mechanics from which Newton's laws can be derived from first principles. The Schrdinger equation is a differential equation, whereas the Feynman integral is an integral of a functional over all possible functions within a space. For example, an integral of F[f(x)] over all possible functions f(x) is the uncountably infinite dimensional integral over all variables $x_0, ..., x_i, ..., x_n$, where "i" is any real number between 0 and n, which is the domain over which $f(x)$ is defined. The dynamics of a quantum system is defined by a "double Feynman integral", which is an integral of a functional of two functions: $F[f(x), g(y)]$, or an integral that has an infinite number of integration variables x_i, and another infinite number of integration variables y_i.

For simple systems (free particle, harmonic oscillator, hydrogen atom, etc.) these Feynman integrals can be calculated analytically, but analytic expressions don't exist for the vast majority of systems. One might imagine discretizing x and y into a *finite* number N of points each and doing the 2N dimensional integral using Monte Carlo, but since the functional integrand is complex valued, and rapidly oscillating, Monte Carlo methods suffer from the famous "sign problem".

With the advent of powerful computers, we can finally calculate these seemingly impossible Feynman integrals by deterministic, numerical methods, which turn out to be perfect for GPU parallelization. We observe factors of 16x speed-up on the GPU compared to the CPU, and ironically, numerical evaluation of Feynman integrals turns out to be easier than solving the Schrdinger differential equation in many cases!

We present applications to quantum computing, chemical reaction rates, and transfer of solar energy in photosynethesis.

Rigorous defect control and the numerical solution of ODEs
John Ernsthausen (McMaster)
Corless and Corliss proposed rigorous defect control of ODE initial value problems over 25 years ago. This talk concerns the implementation and validation of this algorithm. Advances in software and compiler technologies enable efficient, automatic, and rigorous computation of a tight upper bound on the sup-norm for the true defect.

Modern ODE solvers construct a continuously differentiable approximate solution. Evaluating the differential equation at this approximate solution defines the true defect. A defect control approach selects a stepsize such that a measure of the defect is bounded by a user specified tolerance on each step, and, hence, the defect is bounded globally by this tolerance parameter.

Our goal is to know to mathematical certainty that the sup-norm of the true defect is bounded by a user specified tolerance. Given a continuously differentiable approximate solution z of the local ODE initial value problem $x' = f(t, x)$ subject to the initial condition $x(t_i) = x_i$, we evaluate a rigorous polynomial approximation $(T_k, [-d, d])$ of the true defect $\Delta z(t) = z'(t) - f(t, z(t))$; that is, we compute a Taylor model of the true defect. Then, we compute the sup-norm of the degree k polynomial T_k. We then have a tight, mathematically rigorous upper bound on the sup-norm for the true defect over the step. We use the SOLLYA software package to compute rigorous Taylor models and sup-norms.

Mixing LSMC and PDE methods to Price Bermudan Options

David Farahany (Toronto)

In this talk, we develop a mixed least squares Monte Carlo-partial differential equation (LSMC-PDE) method for pricing Bermudan style options in the context of stochastic volatility models. The algorithm is formulated for an arbitrary number of assets and volatility processes, and its probabilistic convergence is established. Our numerical examples focus on the Heston model and we compare the hybrid algorithm with a standard LSMC approach. Using Fourier methods, we are able to derive FFT based solutions, and we demonstrate that our algorithm greatly reduces the variance in the computed prices and optimal exercise boundaries. We also compare the early exercise boundaries and prices computed by our hybrid algorithm with those produced by finite difference methods and find excellent agreement.

This is joint work with Kenneth Jackson and Sebastian Jaimungal.

Comparison of higher order accurate hybridizable and embedded space-time discontinuous Galerkin methods

Tamas Horvath (Waterloo)

The space-time Discontinuous Galerkin (DG) method is an excellent method to discretize problems on deforming domains. This method uses DG to discretize both in the spatial and temporal direction allowing for an arbitrarily high order approximation in space and time. Furthermore, this method automatically satisfies the geometric conservation law which is essential for accurate solutions on time dependent domains. However, the number of unknowns increase significantly when discretizing in 4D space-time. Therefore, we are developing higher-order accurate Hybridizable or Embedded Discontinuous Galerkin (HDG and EDG) methods, because these methods have significantly less degrees of freedom than a standard DG method. We will present some comparisons between these methods.

Learning minimum variance discrete hedging directly from market

Nian Ke (Waterloo)

Option hedging is a critical risk management problem in finance. In the Black-Scholes model, it has been recognized that computing hedging position from the sensitivity of the calibrated model option value

function is inadequate in minimizing variance of the option hedge risk, as it fails to capture the model parameter dependence on the underlying price. In this talk we demonstrate that this issue can exist generally when determining hedging position from the sensitivity of the option function, either calibrated from a parametric model from current option prices or estimated nonparametricaly from historical option prices. Consequently the sensitivity of the estimated model option function typically does not minimize variance of the hedge risk, even instantaneously. We propose a data driven approach to directly learn a hedging function from the market data by minimizing variance of the local hedge risk. Using the S&P 500 index daily option data for more than a decade ending in August 2015, we show that the proposed method outperforms the parametric minimum variance hedging method proposed by Hull and White as well as minimum variance hedging corrective techniques based on stochastic volatility or local volatility models.

Furthermore, we show that the proposed approach achieves significant gain over the implied BS delta hedging for weekly and monthly hedging.

A Data Assimilation Scheme for the One-dimensional Shallow Water Equations
Ramsha Khan (McMaster)
In ocean wave modelling, information on the system dynamics and full initial and/or boundary data is required. When the latter is not fully available the primary objective is to find an optimal estimate of these conditions, using available information. Data Assimilation is a methodology used to optimally integrate observed measurements into a mathematical model, to generate a better estimate of some control parameter, such as the initial condition of the wave, or the sea floor bathymetry. In this study, we considered the shallow water equations in both linear and non-linear form as an approximation for ocean wave propagation, and derived a data assimilation scheme to optimise some distorted form of the initial condition to generate predictions converging to the exact initial data. The error between measurements and observation data was sufficiently minimised across all cases. A relationship was found between the number of measurement points and the error, dependent on the choice of where measurements were taken.

Applying Langevin Based MCMC methods to Radial Velocity Data
Rejean Leblanc (Toronto)

A Variable Step Size Implicit-Explicit Scheme for the Solution of the Poisson-Nernst-Planck Equations
Mary Pugh (Toronto)

Load balancing distributed branch-and bound computations using lowest common ancestors and controlled branching *Srinivas Tamvada (McMaster)*

Developments and Opportunities in Summation-by-Parts Operators

David Zingg (University of Toronto Institute for Aerospace Studies)

Spatial discretizations of partial differential equations that have the summation-by-parts (SBP) property can often be provably stable. This property has recently been generalized to accommodate operators applied as element-type operators, greatly extending the applicability of the concept. In particular, this generalization extends to multiple dimensions such that SBP operators can be developed for unstructured meshes in addition to the traditional tensor-product form applicable only to structured meshes. These generalizations of SBP operators provide numerous opportunities for the development of robust high-order methods for the solution of partial differential equations such as the Navier-Stokes equations governing fluid flow.

Structural analysis of integral-algebraic equations

Reza Zolfaghari (McMaster)

Abstract: We consider the linear integral algebraic equation (IAE) of the general form. Since the Jacobian is singular, the compatibility condition is not always sufficient for consistency. Differentiating some or even all of the equations produces new equations that must also be satisfied by the initial condition. In general it is difficult to determine hidden constraints and hidden compatibility conditions needed for solvability. Also, there are some difficulties in applying discretization methods for integral equations directly to IAEs. We extend the Signature method for DAEs to analyze the structure of an IAE. Our structural approach analyzes the properties of an IAE based on its sparsity pattern and the ν-smoothing property of Volterra integral operators. As a result, we determine which equations and how many times to be differentiated to reveal useful information such as hidden constraints and compatibility conditions.
