Solutions to Test 1

1. c)

$$x_1 = -x_3 - 2x_4 + x_5 + 2 = -s + 3t$$
$$x_2 = x_3 - x_4 - 1 = s + t - 2$$
$$x_3 = s$$
$$x_4 = 1 - t$$
$$x_5 = t$$

2. D)

$$\begin{pmatrix} 2 & 1 & 8 & 1 \\ 0 & -1 & -2 & 1 \\ 1 & 1 & 5 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 1 & 5 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & -1 & -2 & -1 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & 1 & 5 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. c) $(B^TAC)^T$ is 5×7; B^TAC is 7×5

AC is 3×5

so B^T is 7×3 and B is 3×7

4. c)

$$(A - I)^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$A - I = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

5. c) b) is false since A and B may not commute.

6. c)

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

7. A)

$$\begin{pmatrix} 3 & 2 \\ a & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix}$$

We must have $\det\begin{pmatrix} 3 & 2 \\ a & a \end{pmatrix} = 0$ i.e. $a = 0$

and $b \neq 0$.

8. c) Diagonal entries must be non-zero.

9. c) Solve

$$\begin{aligned} a - b + c &= 1 \\ c &= 2 \\ a + b + c &= 5 \end{aligned}$$

to get $b = 2$

10. A).

11. c)

12. E) Interchange row 2 and 3 as well as rows 1 and 4.

13. D) $\det(A^T B C^{-1}) = \dfrac{\det(A)\det(B)}{\det(C)} = -\dfrac{3}{2}$

14. c) $|B| = 6 \begin{vmatrix} a-d & b-e & c-f \\ a+g & b+h & c+i \\ d & e & f \end{vmatrix}$

$= 6 \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$

$= -6\det(A)$. so $\det(A) = -1$

15. D) a) is false - look at $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$