Solutions to Test 2

1. (A) If $u = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ then the first entry of

Au is -2 which is $2\cdot(-1)$ so the eigenvalue is 2.

2. (A) The matrix A has three eigenvalues $1, -1$ and 2
A P which will diagonalize it will have columns that are eigenvectors for these eigenvalues. One checks that the matrix in (A) has columns which are resp. eigenvectors for $2, 1$ and -1.

3. (A) a) The constant is non-zero
b) If P diagonalizes A then $(P^{-1})^T$ diagonalizes A^T.

4. (B) The characteristic polynomial will be $(\lambda^2-1)(\lambda^2+1)$ which is true for (B).

5. (D) One is looking for a probability vector which is an eigenvector for the eigenvalue 1.

6. (C) If the matrix is $A = (a_{ij})$ then a_{11} is the ~~number~~ percentage of baby trees replaced by baby trees given as .1 and so .9 become young trees which is a_{21}; $a_{31} = 0$. Young trees replaced by baby trees is given as .2 so $a_{12} = .2$; $a_{22} = 0$ and $a_{32} = .8$. Old trees replaced by baby trees, a_{13}, is .3 so $a_{23} = 0$ and $a_{33} = .7$.

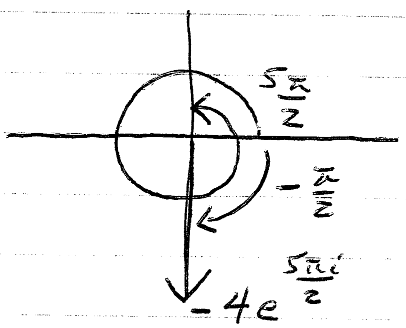
7. (D) $\left.\begin{aligned} x + iy &= 2 \\ (i-1)x - y &= 0 \end{aligned}\right\}$ $y = (i-1)x$ so

$x + i(i-1)x = 2$ or $(1-1-i)x = 2$

so $x = 2i$

and $y = 2i(i-1) = -2(1+i)$

8. (D)



9. (A) and (C) $-8 = 2^3\left(\cos(\pm\pi) + i\sin(\pm\pi)\right)$

so both $2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$ and $2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ are cube roots.

10. (C) b) is false since $e^{2\pi i} = e^0$

11. (D) The plane in question will have the form

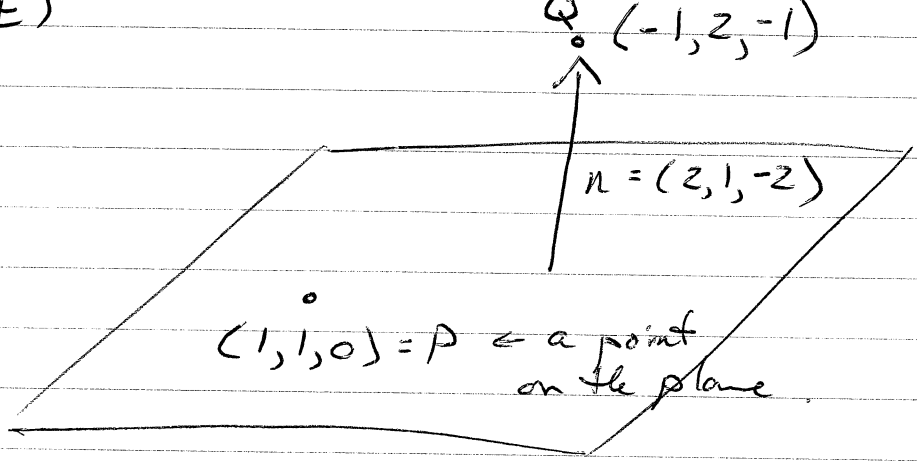
$2x + y - 2z = d$ and $(1,1,-1)$ lies on the plane so $d = 5$.

12. (E) If θ is the angle then $\cos(\theta) = \frac{(1,0,1,1)\cdot(1,1,1,0)}{\sqrt{3}\cdot\sqrt{3}}$

$$= \frac{1}{3}$$

which is not the cosine of any angle listed.

13. (E)



Let $u = \overrightarrow{PQ} = (-2, 1, -1)$ and the distance is

$$\left| \frac{u \cdot n}{\|n\|} \right| = \frac{|(-2,1,-1)\cdot(2,1,-2)|}{\sqrt{9}} = \frac{|-1|}{3} = \frac{1}{3}$$

14. (C) $\left| \begin{vmatrix} 2 & -1 & 3 \\ -1 & -2 & 1 \\ 0 & 2 & -1 \end{vmatrix} \right| = 5$

15. (B) Both are false. In $\mathbb{R}^3$ imagine all vectors orthogonal to a given vector. The dot product will be 0 and the projection will be 0 but the vectors needn't be equal.