Mathematics 1B03 Test 2

Dr. Bradd Hart Nov. 14, 2014

Last Name:______________________________ Initials:__________

Student No.:______________________________

- The test is 50 minutes long.
- The test has 8 pages and 15 multiple-choice questions printed on BOTH sides of the paper.
- Pages 6 through 8 contain no questions and can be used for rough work.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Select the one correct answer for each question and enter that answer onto the scan card provided using an HB pencil. Each question is worth one mark and the total number of marks is 15. There is no penalty for a wrong answer.
- No marks will be given for the work in this booklet. Only the answers on the scan card count for credit. You must submit this test booklet along with your answer sheet.
- You may use a Casio fx-991 MS or MSPlus calculator; no other aids are not permitted.

It is your responsibility to ensure that the answer sheet is properly completed. Your test results depend upon proper attention to these instructions.

The scanner determines your choice of answer by sensing areas of non-reflection. In order for your answer to be read, a heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be seen by the scanner. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 and fill the corresponding bubbles underneath.
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked “1”.

1. The eigenvalues of the matrix

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

are

A) 1, 2, 3, 4 B) 0, 1, 2, 3 C) 1, 2, 3
D) 0, 1, 2 E) None of these

2. Suppose that

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

Which of the following matrices P diagonalizes A i.e. for which P is $P^{-1}AP$ a diagonal matrix?

A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 3 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & 0 \end{pmatrix}$
D) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ E) None of these

3. Suppose that A is a 3×3 matrix with characteristic polynomial $\lambda^3 + 2\lambda^2 + 4\lambda - 3$. Then the trace and determinant of A are

A) 2 and 4 B) -2 and 3 C) -2 and -3
D) 2 and 3 E) None of these

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4. Which of the following vectors is an eigenvector for the following matrix?

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

A) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ B) $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ C) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

D) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ E) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

5. A 4×4 matrix A has eigenvalues $-2, -1, 1$ and 2. Which of these matrices is similar to A?

A) $\begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ B) $\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ C) $\begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i \\ 0 & 0 & -2i & 0 \end{pmatrix}$

D) $\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ E) $\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix}$

6. Given the stochastic matrix $A =$

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/3 \\ 1/4 & 0 & 2/3 \end{pmatrix}$$

which of the following is the steady-state or equilibrium vector for the Markov chain associated with A?

A) $\begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}$ B) $\begin{pmatrix} 3/11 \\ 4/11 \\ 4/11 \end{pmatrix}$ C) $\begin{pmatrix} 4/11 \\ 3/11 \\ 4/11 \end{pmatrix}$

D) $\begin{pmatrix} 4/11 \\ 4/11 \\ 3/11 \end{pmatrix}$ E) None of these

continued ...

7. Which of the following statements are true?

   (a) The column vectors of a transition matrix are probability vectors.

   (b) Every stochastic matrix has 1 as an eigenvalue.

   A) Both B) Neither C) (a) D) (b)

8. Compute the determinant of the following matrix:

$$\begin{pmatrix} i & 0 & 0 \\ 0 & 1-i & 1 \\ 0 & i & 1+i \end{pmatrix}$$

   A) $1+2i$ B) $2+i$ C) $1-2i)$

   D) $-1+2i$ E) None of these

9. Compute $\dfrac{1+2i}{1-3i}$

   A) $1-\dfrac{2i}{3}$ B) $-\dfrac{1}{2}+\dfrac{i}{2}$ C) $-\dfrac{1}{2}-\dfrac{i}{2}$

   D) $\dfrac{1}{2}+\dfrac{i}{2}$ E) None of these

10. Which of the following is a fifth root of 32?

   A) $2(\cos(\pi/5)+i\sin(\pi/5))$
   B) $2(\cos(\pi/5)-i\sin(\pi/5))$
   C) $2(\cos(2\pi/5)+i\sin(2\pi/5))$
   D) $-2(\cos(2\pi/5)+i\sin(2\pi/5))$
   E) None of these

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11. Which of the following statements are true?

    (a) $z\bar{z} = |z|^2$ for all complex numbers z.

    (b) $e^{i\theta} = e^{i(\theta+2\pi)}$ for all θ.

    A) Both B) Neither C) (a) D) (b)

12. If for $u, v \in \mathbb{R}^n$ we have $\|u+v\| = 5$ and $\|u\| = 3$ and $\|v\| = 4$ then $u \cdot v =$

    A) 2 B) -2 C) $\frac{5}{12}$ D) 0 E) None of these

13. Find the distance from the point $(1, 2, 3)$ to the plane $x - y + 2z = 2$.

    A) $\frac{5}{\sqrt{6}}$ B) $\frac{-3}{\sqrt{6}}$ C) $\frac{5}{\sqrt{2}}$ D) $\frac{3}{\sqrt{2}}$ E) None of these

14. Find the area of the parallelogram in $\mathbb{R}^2$ with edges parallel to $(2, 3)$ and $(1, 2)$.

    A) -1 B) 1 C) 4 D) 3 E) None of these

15. Which of the following statements are true?

    (a) For $u, v, w \in \mathbb{R}^3$, $u \times (v \times w)$ lies in the plane determined by v and w.

    (b) If $u, v \in \mathbb{R}^n$ are orthogonal then $\|u + 2v\|^2 = \|u\|^2 + 4\|v\|^2$.

    A) Both B) Neither C) (a) D) (b)

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