Linear algebra, Math 1B3

Bradd Hart

Sept. 5, 2014

Short-term outline

- Review linear equations, section 1.1
- Introduce Gaussian elimination, section 1.2
- Introduce matrices and matrix algebra, sections 1.3 - 1.4

Linear equations

- A linear equation in n unknowns or variables has the form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$$

  where the x_i's are the unknowns and the a_i's and the b are numbers.
- We may use letters like x, y or z for the variables when we have only a few.
- If when you substitute $x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$ into the linear equation above and the left hand side equals the right hand side, we say that this is a solution to that linear equation. We call $(s_1, \ldots, s_n)$ an ordered n-tuple.

Systems of linear equations

A finite set of linear equations is called a system of linear equations; in general a linear system looks like this:

$$\begin{array}{ccccc} a_{11}x_1 + a_{12}x_2 + & \ldots & +a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + & \ldots & +a_{2n}x_n & = & b_2 \\ \vdots & \ddots & \vdots & & \\ a_{m1}x_1 + a_{m2}x_2 + & \ldots & +a_{mn}x_n & = & b_m \end{array}$$

A solution to a linear system is an ordered n-tuple which is simultaneously a solution to each equation.

Augmented matrices

For the linear system

$$\begin{array}{cccccc} a_{11}x_1 + a_{12}x_2 + & \dots & +a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + & \dots & +a_{2n}x_n & = & b_2 \\ \vdots & \ddots & \vdots & & \\ a_{m1}x_1 + a_{m2}x_2 + & \dots & +a_{mn}x_n & = & b_m \end{array}$$

the augmented matrix is

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \ddots & \vdots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{pmatrix}$$

Elementary row operations

To solve a linear system one performs a series of algebraic operations:

- Multiply an equation by a non-zero constant
- Add a constant multiple of one equation to another
- Interchange equations

The main fact is that doing these operations doesn't change the set of solutions of the linear system.

The corresponding *elementary row operations* on an augmented matrix are:

- Multiply a row by a non-zero constant
- Add a constant multiple of one row to another
- Interchange two rows