The cross product

Definition

Suppose that $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are two vectors in $\mathbb{R}^3$. We define the cross product of u with v, written $u \times v$, as

$$(u_2 v_3 - v_2 u_3, v_1 u_3 - u_1 v_3, u_1 v_2 - v_1 u_2)$$

- $u \times v$ can also be described geometrically as a vector which is both orthogonal to u and v, has length $||u||||v|| \sin(\theta)$ where θ is the angle between u and v, and is oriented according to the right-hand rule.

Area, volume and the determinant

- The area of the parallelogram determined by two vectors $u, v \in \mathbb{R}^3$ is $\|u \times v\|$.
- So if $u = (u_1, u_2)$ and $v = (v_1, v_2)$ then the area of the parallelogram determined by u and v is the absolute value of

$$\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

- The area of the triangle determined by two vectors $u, v \in \mathbb{R}^3$ is $\|u \times v\|/2$. These formulas even work in $\mathbb{R}^2$.
- The volume of the parallelepiped determined by three vectors $u, v, w \in \mathbb{R}^3$ is $|u \cdot (v \times w)|$. So if A is the 3 x 3 matrix with rows given by u, v and w then the area of this parallelepiped is $|\det(A)|$.

The test

- The second test is scheduled for Nov. 14 at 10:30 am (that is class time). The test will be 50 minutes.
- If your surname is in the range A - O then you will write the test in T28/001; P - Z will write in T29/101.
- The test will be multiple choice and you will need to bring an HB pencil. You will be allowed to have the McMaster approved Casio fx-991 MS but no other aids. Please bring your ID card with you to the test.
- The test will cover sections 5.1 - 5.2, 5.5, 3.1 - 3.5 and 10.1 - 10.3 from the 9th edition. I will post additional problems.
- There is a practice test for Test 2. Please try the practice test once you have studied for the test; I will post the solutions on Monday.
- There is a review class which will be run by Matt on Thursday, Nov. 13, 5:30 - 7:30 in HH 302; the class Wed. Nov. 12 will also be review.

Vector spaces

Definition

Suppose that V is a non-empty set, $+$ is function on pairs from V and for every $k \in \mathbb{R}$ and $u \in V$, ku is defined. We say that V together with these operations defines a vector space if the following axioms are satisfied for all $u, v, w \in V$ and $k, m \in \mathbb{R}$:

Vector spaces, cont'd

1. $u + v \in V$.
2. $u + v = v + u$
3. $u + (v + w) = (u + v) + w$
4. there is a $0 \in V$ such that $u + 0 = u$ for all $u \in V$
5. there is a $-u \in V$ such that $u + (-u) = 0$
6. $ku \in V$
7. $k(u + v) = ku + kv$
8. $(k + m)u = ku + mu$
9. $k(mu) = (km)u$
10. $1u = u$