A quick review

- If V is a vector space and X is a subset of V then there is a subspace W containing X with the property that if any other subspace W' contains X then $W \subseteq W'$. This subspace is called the span of X.
- The span of X is made up of all linear combinations of elements of X.
- We say that v depends on X if v is in the span(X).
- v depends on X iff v can be expressed as a linear combination of vectors from X.

Definition

A subset S of a vector space V is linearly independent if whenever $v_1, \ldots, v_n \in S$ are distinct and

$$k_1 v_1 + \ldots + k_n v_n = 0$$

then it must be that

$$k_1 = \ldots = k_n = 0$$

S is said to be linearly dependent if it is not linearly independent.

A couple of important facts

Fact

If V is a vector space and $S \subseteq V$ then S is linearly dependent iff for some $v \in S$, v depends on $S \setminus \{v\}$ i.e. v can be written as a linear combination of vectors from $S \setminus \{v\}$.

Theorem

If S is a set of more than n vectors from $\mathbb{R}^n$ then S is linearly dependent.

Basis

Definition

If V is a vector space and S is a subset of V then S is a basis for V if

1. S is linearly independent and
2. S spans V.

Theorem

If S is a basis for V then every vector $v \in V$ can be expressed in the form

$$v = k_1 v_1 + k_2 v_2 + \ldots + k_n v_n$$

for distinct $v_1, \ldots, v_n \in S$ in exactly one way.