The Putnam competition

- The William Lowell Putnam competition is a university-level mathematics competition held annually for undergraduate students at North American universities.
- More information can be found at math.scu.edu/putnam or on the undergraduate page of the department's website.
- This year's competition will occur on Dec. 6. If you are interested in participating or learning more, drop by Monday and/or send me email.
- Training sessions for this year's Putnam competition will occur on Mondays at 12:30 in HH 410 beginning Sept. 22.

Matrix multiplication

- If A is an $m \times k$ matrix and B is a $k \times n$ matrix then we can multiply A by B forming AB (the order is important). AB is an $m \times n$ matrix.
- For i and j such that $1 \leq i \leq m$ and $1 \leq j \leq n$ then we need to specify the ij entry of AB.
- Suppose the i^{th} row of A and j^{th} column of B are

$$\begin{pmatrix} a_{i1} & a_{i2} & \ldots & a_{ik} \end{pmatrix} \text{ and } \begin{pmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{kj} \end{pmatrix}$$

Then the ij entry of AB is

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{ik}b_{kj}$$

Back to linear systems

- Suppose we have the linear system

$$\begin{array}{ccccc} a_{11}x_1 + a_{12}x_2 + & \ldots & +a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + & \ldots & +a_{2n}x_n & = & b_2 \\ \vdots & \ddots & \vdots & & \\ a_{m1}x_1 + a_{m2}x_2 + & \ldots & +a_{mn}x_n & = & b_m \end{array}$$

- Let A, x and b be the matrices

$$\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \ddots & \vdots & \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ and } \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

- Then the linear system can be written using matrices as $Ax = b$.

The transpose and the trace

- If A is an $m \times n$ matrix then the transpose of A, written A^T is the $n \times m$ matrix whose ij entries are $(A)_{ji}$.
- If A is an $n \times n$ matrix i.e. a square matrix, then the trace of A, written $tr(A)$ is

$$a_{11} + \ldots + a_{nn}$$

Properties of Matrices

Theorem (1.4.1)

Assume that A, B and C are matrices for which the following operations make sense. Then

(c) $A(BC) = (AB)C$; *associativity of matrix multiplication*

(d) $A(B + C) = AB + AC$; *left distributivity*

(e) $(B + C)A = BA + CA$; *right distributivity*

Some special matrices

- A matrix with only zero entries is called a zero matrix.
- If A is an $n \times n$ matrix with 1's on the diagonal and 0's everywhere else than A is called the identity matrix and often written I_n or sometimes just I if we remember how large it is.

$$I_n = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Invertible matrices

Definition

We say that a square matrix A is invertible if there is a square matrix B of the same size as A such that $AB = I$ and $BA = I$. We call B an inverse of A.

- If A is a square matrix and not invertible we say it is singular and sometimes we refer to invertible matrices as nonsingular.
- If a square matrix A has an inverse then that inverse is unique and we write A^{-1}. So A^{-1} is **the** inverse of A.

Properties of invertible matrices

Theorem (1.4.6)

Suppose that A and B are invertible $n \times n$ matrices. Then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Theorem (1.4.7)

Suppose that A is an invertible matrix. Then

(a) *A^{-1} is invertible and $(A^{-1})^{-1} = A$.*

(b) *For any natural number n, A^n is invertible and $(A^n)^{-1} = (A^{-1})^n$.*