MATH 1B03 Day Class Final Exam
Bradd Hart, Dec. 13, 2013

Name: ______________________________ **ID #:** ______________

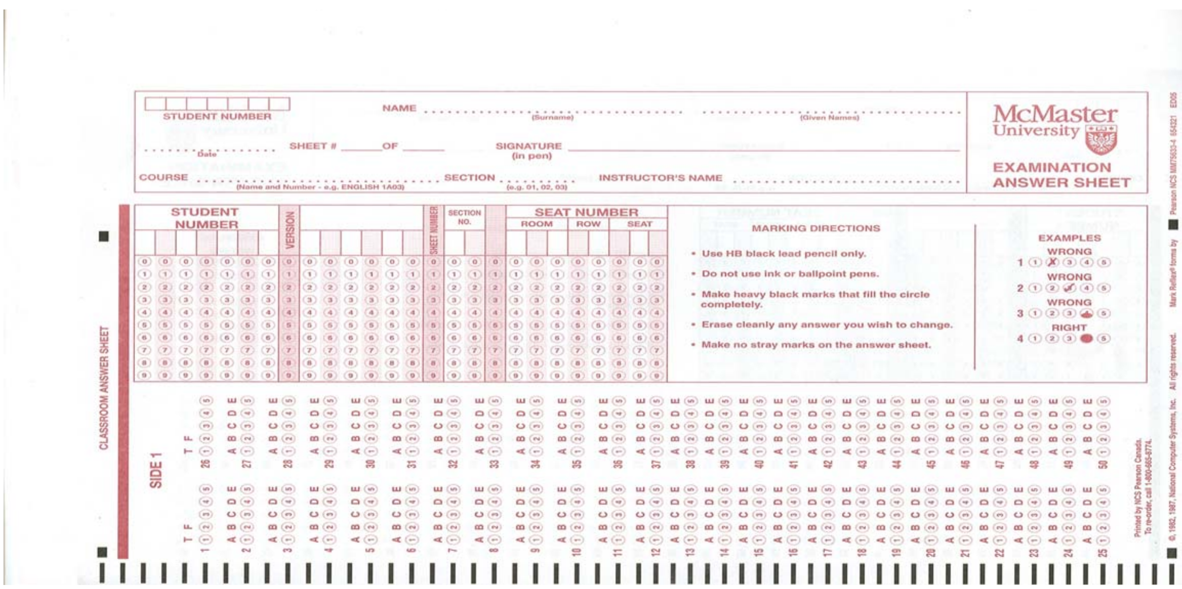
- The exam is 3 hours long.
- The exam has questions on page 2 through 22; there are 40 multiple-choice questions printed on BOTH sides of the paper.
- Pages 23 to 28 contain no questions and can be used for rough work.
- You are responsible for ensuring that your copy of the exam is complete. Bring any discrepancies to the attention of the invigilator.
- Select the one correct answer for each question and enter that answer onto the answer sheet provided using an HB pencil.
- Each question is worth one mark and the total number of marks is 40.
- There is no penalty for a wrong answer.
- No marks will be given for the work in this booklet. Only the answers on the answer sheet count for credit. You must submit this test booklet along with your answer sheet.
- You may use a Casio fx-991 calculator; no other aids are not permitted.
- **Good luck**

OMR EXAMINATION - STUDENT INSTRUCTIONS

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, section number and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only **ONE** choice from the alternatives (1,2,3,4,5 or A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked "1".


STUDENT NUMBER
NAME (Surname) (Given Names)
Date SHEET # OF SIGNATURE (in pen)
COURSE (Name and Number - e.g. ENGLISH 1A03) SECTION (e.g. 01, 02, 03) INSTRUCTOR'S NAME
McMaster University
EXAMINATION ANSWER SHEET
STUDENT NUMBER
VERSION
SHEET NUMBER
SECTION NO.
SEAT NUMBER
ROOM ROW SEAT
MARKING DIRECTIONS
• Use HB black lead pencil only.
• Do not use ink or ballpoint pens.
• Make heavy black marks that fill the circle completely.
• Erase cleanly any answer you wish to change.
• Make no stray marks on the answer sheet.
EXAMPLES
WRONG
WRONG
WRONG
RIGHT
CLASSROOM ANSWER SHEET
SIDE 1

Continued on page 3

1. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the following matrix in reduced row echelon form.

$$\begin{pmatrix} 1 & -5 & 0 & 0 & 9 & -9 \\ 0 & 0 & 1 & 0 & -9 & 5 \\ 0 & 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The solution to the system of linear equations is:

A) $$\begin{aligned} x_1 &= -9+5s-9t \\ x_2 &= s \\ x_3 &= 5+9t \\ x_4 &= 3-5t \\ x_5 &= t \end{aligned}$$

B) $$\begin{aligned} x_1 &= 9-5s+9t \\ x_2 &= s \\ x_3 &= -5-9t \\ x_4 &= -3+5t \\ x_5 &= t \end{aligned}$$

C) $$\begin{aligned} x_1 &= 5r-9s+9t \\ x_2 &= r \\ x_3 &= 9s-5t \\ x_4 &= 5s-3t \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

D) $$\begin{aligned} x_1 &= -9+5s-9t \\ x_2 &= s \\ x_3 &= 5s+9t \\ x_4 &= 3s-5t \\ x_5 &= t \end{aligned}$$

E) $$\begin{aligned} x_1 &= -9 \\ x_2 &= 0 \\ x_3 &= 5 \\ x_4 &= 3 \\ x_5 &= 0 \end{aligned}$$

2. Which of the following statements are true?

1. If a linear system has more equations than unknowns then it must have infinitely many solutions.
2. It is possible for a system of linear equations to have exactly two solutions.

A) Both B) Neither C) (1) D) (2)

Continued on page 4

3. Which of the following statements are true?

1. If A is a square matrix with two identical columns then $\det(A) = 0$.
2. For every square matrix A and every scalar c, $\det(cA) = c\det(A)$.

A) Both B) Neither C) (1) D) (2)

4. Solve the following matrix equation for A:

$$(A^T + 3I)^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

A) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix}$ C) $\begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix}$

D) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ E) None of these

Continued on page 5

5. What are the solutions to

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} ?$$

A) $\begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ B) $\begin{pmatrix} 0 \\ 2 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ C) $\begin{pmatrix} 0 \\ 2 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 1 \\ -2 \end{pmatrix}$

D) $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ E) $\begin{pmatrix} 1 \\ 0 \\ 1 \\ -2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ -2 \\ 0 \end{pmatrix}$

6. Which of the following statements are true?

1. The product of elementary matrices is elementary.
2. Every square matrix can be written as the product of elementary matrices.

A) Both B) Neither C) (1) D) (2)

Continued on page 6

7. Compute the following determinant:

$$\begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

A) 18 B) -18 C) 6 D) -6 E) None of these

8. For what value of k is the matrix A not invertible, where

$$A = \begin{pmatrix} k & 0 & 1 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 2 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{pmatrix}?$$

A) -1 B) 1 C) 2 D) -2 E) 3

Continued on page 7

9. Suppose that A and B are both $n \times n$ matrices, $\det(A) = 3$ and $\det(A^{-1}B) = 4$. Then $\det(BA)$ is:

A) 4/3 B) 36 C) 12 D) 4 E) cannot be determined

10. Which of the following is not equivalent to the others for an n x n matrix A?

A) $\det(A) = 0$
B) The reduced row echelon form of A is I.
C) A can be expressed as a product of elementary matrices.
D) The linear system $Ax = b$ has a solution for all b.
E) The dimension of the null space of A is 0.

Continued on page 8

11. A 3x3 matrix A has a characteristic polynomial $\lambda^3 - 6\lambda^2 + 13\lambda - 18$. The determinant of A is

A) -6 B) 6 C) 18 D) -18 E) None of these

12. Consider the 3 vectors in P_3, the vector space of polynomials of degree less than or equal to 3.

$$1 - x, 1 + 3x + x^2 \text{ and } 3 - 2x + x^3$$

Which of the following statements is correct?

A) The vectors are not linearly independent, and do not form a basis for P_3
B) The vectors are not linearly independent, and form a basis for P_3
C) The vectors are linearly independent, and form a basis for P_3
D) The vectors are linearly independent, and do not form a basis for P_3

Continued on page 9

13. The area of a triangle in $\mathbb{R}^3$ with vertices $(1,0,-1),(2,1,3)$ and $(0,2,-1)$ is

A) 11 B) $\frac{11}{2}$ C) $\sqrt{89}$ D) $\frac{\sqrt{89}}{2}$ E) None of these

14. The cosine of the angle between $(1,0,2,1)$ and $(1,2,0,1)$ in $\mathbb{R}^4$ is

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) 2 D) 0 E) None of these

Continued on page 10

15. The projection of u onto v in $\mathbb{R}^5$ where

$$u = (1, 0, 1, 0, 2) \text{ and } v = (0, 1, 1, 1, 1)$$

is

A) $\frac{1}{2}(1, 0, 1, 0, 2)$ B) $\frac{3}{4}(1, 0, 1, 0, 2)$ C) $\frac{1}{2}(0, 1, 1, 1, 1)$

D) $\frac{3}{4}(0, 1, 1, 1, 1)$ E) None of these

16. Consider the line in $\mathbb{R}^4$ which passes through the points $(1, 0, 0, -1)$ and $(2, 1, 0, 1)$. The point on this line closest to the point $(2, 2, 2, 2)$ is

A) $\frac{1}{2}(3, 3, 0, 6)$ B) $\frac{1}{2}(10, 7, 0, 11)$ C) $(3, 3, 3, 3)$

D) $\frac{1}{2}(5, 3, 0, 4)$ E) None of these

Continued on page 11

17. If $z = e^{i\frac{\pi}{3}}$ then $z^{1000} =$

A) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ B) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ C) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

D) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ E) None of these

18. The modulus of $\frac{1+3i}{1-i}$ is

A) $\sqrt{10}$ B) $\sqrt{5}$ C) $-1+2i$ D) $1+i$ E) None of these

Continued on page 12

19. Which of the following statements are always true?

1. There is a vector space with exactly two elements.
2. The union of two subspaces of a vector space is a subspace.

A) Both B) Neither C) (1) D) (2)

20. The eigenvalues of the following matrix

$$\begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$$

are

A) 5, -1 and -2 B) 3 and -4 C) 0, 3 and -4
D) -3 and 4 E) 0, -3 and 4

Continued on page 13

21. Which of the following vectors are eigenvectors for

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix}?$$

1. $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ 2. $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ 3. $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

A) All of them B) 1 only C) 1 and 2 only
D) 2 and 3 only E) 1 and 3 only

22. Suppose that A is the complex matrix

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 1 \end{pmatrix}$$

Determine the eigenvalues of A - remember that they may be complex.

A) $1, i, -i$ B) $1, -1, i$ C) $0, 1, 2$
D) $-1, 0, 1$ E) $0, i, -i$

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23. What is the dimension of the subspace of M_{22}, the vector space of 2x2 matrices, spanned by

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix}?$$

A) 0 B))1 C) 2 D) 3 E) 4

24. Which of the following statements are true?

1. There is a set of 16 vectors that spans $\mathbb{R}^{14}$.
2. There is a set of 11 vectors that is linearly independent in $\mathbb{R}^{12}$.

A) Both B) Neither C) (1) D) (2)

Continued on page 15

25. Which of the following statements are true?

1. If E is an elementary $m \times m$ matrix and A is an $m \times n$ matrix then the row space of A and EA are the same.
2. A matrix with linearly independent row vectors and linearly independent column vectors must be square.

A) Both B) Neither C) (1) D) (2)

26. A basis for the row space of the matrix

$$\begin{pmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ -2 & 9 & 2 & -4 & -5 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \end{pmatrix}$$

from among the row vectors is

A) $(1, -3, 2, 2, 1)$ and $(0, 3, 6, 0, -3)$
B) $(1, -3, 2, 2, 1), (0, 3, 6, 0, -3)$ and $(-2, 9, 2, -4, -5)$
C) $(1, -3, 2, 2, 1), (2, -3, -2, 4, 4)$ and $(3, -6, 0, 6, 5)$
D) $(1, -3, 2, 2, 1)$ and $(2, -3, -2, 4, 4)$
E) $(1, -3, 2, 2, 1), (0, 3, 6, 0, -3)$ and $(2, -3, -2, 4, 4)$

Continued on page 16

27. Suppose that the matrix A is

$$\begin{pmatrix} 1 & 3 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

Which of the following matrices diagonalizes A?

A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 5 & 1 \\ 0 & -3 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ C) $\begin{pmatrix} 1 & -5 & -1 \\ 0 & -3 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

D) $\begin{pmatrix} 1 & 0 & 0 \\ 5 & -3 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ E) A cannot be diagonalized.

28. Suppose that V is a 3 dimensional vector space and v_1, v_2, v_3 and v_4 are 4 vectors from V that span V. Moreover suppose that $v_1 + v_2 + v_3 = 0$. Which of the following is a basis for V?

A) $\{v_1, v_2\}$
B) $\{v_1, v_2, v_3\}$
C) $\{v_2, v_3\}$
D) $\{v_1, v_2, v_4\}$
E) There is insufficient information to decide.

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29. Which of the following is a subspace of the indicated vector space?

1. All functions f such that $f(0) = 0$ in the vector space of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$.
2. All upper triangular $n \times n$ matrices in the vector space M_{nn}.
3. All invertible matrices in the vector space M_{nn}.

A) All of them B) None of them C) 1 and 3
D) 1 and 2 E) 2 and 3

30. Which of the following statements are true?

1. Every linearly dependent set contains the zero vector.
2. If V is a 4 dimensional vector space then every subset of V with 5 vectors is linearly dependent.

A) Both B) Neither C) (1) D) (2)

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31. Which of the following sets is linearly independent in the indicated vector space?

1. The functions $\{1, e^x, e^{2x}\}$ in the vector space of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$.
2. The vectors $\{(1,0,3),(2,1,2),(-1,0,1)\}$ in $\mathbb{R}^3$.

A) Both B) Neither C) (1) D) (2)

32. If one uses the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthogonal basis where

$$u_1 = (1,1,1), u_2 = (-1,1,0) \text{ and } u_3 = (1,2,1)$$

then the first two vectors one obtains are just u_1 and u_2 (since they are orthogonal). The third vector from the Gram-Schmidt process is

A) $\frac{4}{3}(1,1,1)$ B) $\frac{1}{2}(-1,1,0)$ C) $\frac{1}{3}(-1,2,1)$

D) $\frac{1}{6}(1,1,-2)$ E) $\frac{1}{2}(3,3,2)$

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33. If A is a 5 x 7 matrix and the number of free variables in the solution of $Ax = 0$ is 3 then the rank of A is

A) 1 B) 2 C) 3 D) 4 E) 5

34. Which of the following is an orthonormal basis for $\mathbb{R}^4$?

A) $(1, 0, 0, 0), (0, 1, 0, 0)$ and $(0, 0, 1, 0)$
B) $(1, -1, 0, 0), (1, 1, 0, 0), (0, 0, 1, 1)$ and $(0, 0, 1, -1)$
C) $(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0)$ and $(1, 1, 1, 1)$
D) $\frac{1}{\sqrt{2}}(1, -1, 0, 0), \frac{1}{\sqrt{2}}(1, 1, 0, 0), \frac{1}{\sqrt{2}}(0, 0, 1, 1)$ and $\frac{1}{\sqrt{2}}(0, 0, 1, -1)$
E) $(1, 0, 0, 0), \frac{1}{\sqrt{2}}(1, 1, 0, 0), \frac{1}{\sqrt{3}}(1, 1, 1, 0)$ and $\frac{1}{2}(1, 1, 1, 1)$

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35. Suppose that we have three vectors in $\mathbb{R}^3$

$$u_1 = (1, 1, 1), u_2 = (0, 1, 1) \text{ and } u_3 = (0, 0, 1)$$

$\{u_1, u_2, u_3\}$ form a basis of $\mathbb{R}^3$ and the vector $v = (1, 0, 1)$ can be expressed as a linear combination of u_1, u_2 and u_3. If $v = k_1u_1 + k_2u_2 + k_3u_3$ then (k_1, k_2, k_3) is

A) (1,0,0) B) (1,-1,1) C) (1,1,1)
D) (0,1,1) E) (0,0,1)

36. If W_1 and W_2 are two different 3 dimensional subspaces of a vector space V and the union of W_1 and W_2 spans V then the dimension of V is

A) 3 or 6 B) 3, 4, 5 or 6 C) 5 or 6
D) 6 E) 4, 5 or 6

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37. Which of the following are bases for the vector space of 2x2 symmetric matrices?

1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

A) All of them B) None of them C) 1 and 2
D) 2 E) 2 and 3

38. A person does not want to be a slave to fashion when picking their sock colour for the day but nonetheless has a preference for white socks. They decide to wear white, black or red socks and never wear the same colour socks on consecutive days. On the day after they wear white socks, they choose black or red with equal chance. However, on the day after wearing red or black socks they are twice as likely to wear white than any other colour. In the long run, what amount of time do they wear white socks?

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{5}$ E) Cannot be determined

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39. Suppose that we consider the set V of all invertible $n \times n$ matrices and the binary operation $\oplus$ defined by $A \oplus B = AB$, ordinary matrix multiplication. Which of the following statements is false?

A) V is closed under the operation $\oplus$
B) for all $A, B \in V$, $A \oplus B = B \oplus A$
C) for all $A, B, C \in V$, $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
D) there is a $0 \in V$ such that for all $A \in V, A \oplus 0 = A$
E) Using 0 from part D, for every $A \in V$ there is a $B \in V$ such that $A \oplus B = 0$

40. The name of your favourite linear algebra professor is

A) Dr. Hart B) Professor Hart C) Bradd Hart
D) Bradd E) All of these

END OF TEST QUESTIONS

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END OF TEST PAPER