Test 1 Solutions

1. B)

2. C)

3. $B(AB)^{-1}AC^{-1}(D^{-1}C^{-1})^{-1}D^{-1} = BB^{-1}A^{-1}AC^{-1}CDD^{-1} = I$

so E)

4. $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ so $A = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$, B)

5. trace is additive but not multiplicative

$0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ but $0 = tr(0) \neq 1 \cdot 1$.

C)

6. D)

7. B): A is invertible iff $\det(A) \neq 0$ iff the reduced row-echelon form of A is I. So a) and c) are equivalent ~~to~~ to invertibility.

$Ax = 0$ always has a solution; $x = 0$. This statement is only equivalent to "A is invertible" if you say $x = 0$ is the only solution.

8. $$\begin{vmatrix} \lambda-1 & -2 & -1 \\ 0 & \lambda-1 & 0 \\ 2 & 4 & \lambda+2 \end{vmatrix} = (\lambda-1)^2(\lambda+2) + 2(\lambda-1)$$

$$= (\lambda-1)\big((\lambda-1)(\lambda+2)+2\big)$$

$$= \lambda(\lambda-1)(\lambda-2)$$

So this is invertible if $\lambda \neq 0, 1$ or 2 ; E).

9. D) : $a - b + c = 6$ so $a-b=4$ and $a=3$
$c = 2$, $a+b=2$
$a+b+c=4$

10. c) : b) is false $(I - I = 0)$.

11. $$\begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = -(3-4+1-6)$$

$= 6$; D)

12. D) is the only possible answer since it is 2×3

13. A^TA is always symmetric
$(A^{-1})^T = (A^T)^{-1} = (-A)^{-1} = -A^{-1}$ so not symmetric.
$(A^2)^T = (A^T)^2 = (-A)^2 = A^2$ so symmetric.

D)

14. $\det(B) = (-1)(-3)\det(A) = 3\det(A)$; B).

15. $\det(B^{-1}AB) = \dfrac{1}{\det(B)} \det(A)\det(B) = \det(A)$

adj(A) is the transpose of the cofactor matrix so it is $n \times n$.

A).