Mathematics 2R3 Test 1

Dr. Bradd Hart — Oct. 9, 2019

Last Name:______________________________ Initials:__________

Student No.:______________________________

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a McMaster standard Casio fx-991 MS or MS Plus calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

Good Luck!

Score

Question	1	2	3	4	5	Total
Points	5	5	5	5	5	25
Score						

continued ...

1. (5 marks) Put your answer in the space provided for each part.

   (a) Every complex $n \times n$ matrix has an eigenvalue. True or false.

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   (b) Every linearly independent set of n vectors in $\mathbf{R}^n$ is a basis for $\mathbf{R}^n$. True or false.

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   (c) There are 4 polynomials in the vector space P_5 of real polynomials of degree less than or equal to 5 that span P_5. True or false.

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   (d) If $u = (i, -i)$ and $v = (-i, -i)$ in $\mathbf{C}^2$ then $u \cdot v =$ __________

   (e) Find the eigenvalues of the following matrix by inspection.

$$\begin{pmatrix} 3-i & 0 & 0 \\ -1 & i & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

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2. (a) (3 marks) Find all complex numbers z such that $z^4 = 1 - i$.

   (b) (2 marks) Suppose V is a complex inner product space and $u, v \in V$ such that $\langle u, u \rangle = 1$, $\langle u, v \rangle = i$ and $\langle v, v \rangle = 1$. Compute $\|u + v\|$.

continued . . .

3. (5 marks) Suppose A is the matrix

$$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors for A in $\mathbf{C}^2$.

continued . . .

4. In the inner product space of continuous functions on $[-1, 1]$ with inner product given by

$$\langle f, g\rangle = \int_{-1}^{1} fg\,dx$$

(a) (3 marks) use the Gram-Schmidt process applied to the linearly independent set $\{1, x, x^2\}$ to form an orthogonal set; you do not have to normalize the vectors.

(b) (2 marks) If f and g are two continuous functions on $[-1, 1]$, explain why

$$\left(\int_{-1}^{1} (f+g)^2 dx\right)^{1/2} \leq \left(\int_{-1}^{1} f^2 dx\right)^{1/2} + \left(\int_{-1}^{1} g^2 dx\right)^{1/2}.$$

continued ...

5. Suppose that W is a subspace of an inner product space V.

   (a) (2 marks) Define what is meant by the orthogonal complement, $W^{\perp}$, of W.

   (b) (3 mark) Prove that $W^{\perp}$ is a subspace of V.

THE END