Eigenvalues

Definition

Suppose that A is an $n \times n$ complex matrix, λ is a scalar and $x \in \mathbb{C}^n$ is non-zero such that

$$Ax = \lambda x$$

Then λ is called an eigenvalue of A and x is called an eigenvector.

Eigenvalues, cont'd

Theorem

If A is an $n \times n$ matrix and λ is a scalar then the following are equivalent:

1. *λ is an eigenvalue of A.*
2. *The system of linear equations $(\lambda I - A)x = 0$ has non-trivial solutions.*
3. *There is a non-zero $x \in \mathbb{C}^n$ such that $Ax = \lambda x$.*
4. *λ is a solution to the characteristic equation $\det(\lambda I - A) = 0$.*

Eigenvalues, cont'd

Theorem

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1. *λ is an eigenvalue of A.*
2. *The system of linear equations $(\lambda I - A)x = 0$ has non-trivial solutions.*
3. *There is a non-zero $x \in \mathbb{C}^n$ such that $Ax = \lambda x$.*
4. *λ is a solution to the characteristic equation $\det(\lambda I - A) = 0$.*

Definition

If λ is an eigenvalue for A, an $n \times n$ matrix, then the set of all x such that $Ax = \lambda x$ forms a subspace of $\mathbb{C}^n$ which is called the eigenspace of A corresponding to λ.

Diagonalizability

Definition

A square matrix A is called diagonalizable if there is an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. P is said to diagonalize A.

Theorem (5.2.1)

The following are equivalent for an $n \times n$ matrix A:

1. *A is diagonalizable.*
2. *A has n linearly independent eignevectors.*