Math 3CY3, Test 1

Bradd Hart, Oct. 4, 2018

There are 5 questions and each question is worth 5 marks; the test will be graded out of 20. Do as many questions as you can in the allotted time.

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (a) Find the multiplicative inverse of 3 modulo 10.
   (b) Find the last two digits of 1413^{162}.
   (c) Is the following matrix invertible mod 26 and if so, find its inverse?

$$\begin{pmatrix} 1 & 6 \\ 2 & 10 \end{pmatrix}$$

2. If the word "dont" is encrypted using a Hill cipher with a 2×2 matrix as "ELNI", find the encryption matrix and encode the word "hi".

3. Suppose that two English generals were known to be communicating using a character substitution cipher. Explain what measures you could take in order to discover what they were saying in their communications.

4. Prove if that is a, b, q and r are integers such that $b = qa + r$ and $r \neq 0$ then $\gcd(a,b) = \gcd(a,r)$.

5. Use the Euclidean algorithm to determine the greatest common divisor of 418 and 165 and then use the extended Euclidean algorithm to write the gcd in the form $418x + 165y$ for integers x and y.