Math 3EE3, Test 1
Bradd Hart, Feb. 10, 2015

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (5 marks) Give examples of the following (justification is not required):

    (a) A finite ring that is not a field.

    (b) An integral domain that is not a field.

    (c) A division ring that is not a field.

    (d) A ring without a 1.

2. (5 marks) Determine if the following polynomials are irreducible over the given fields:

    (a) $x^4 + 3x^2 + 6x + 6$ over Q

    (b) $x^3 + x^2 + 1$ over Z_3

    (c) $x^4 - 2$ over $Q[\sqrt{2}]$

3. (5 marks) Prove that if R is a commutative ring with 1 and I is a maximal ideal then R/I is a field.

4. (5 marks) Determine all the ideals in the following rings:

    (a) Z_{17}

    (b) Z_{16}

    (c) $M_2(Z_2)$

5. (5 marks) Show that $Q[\sqrt{3}] = \{a + b\sqrt{3} : a, b \in Q\}$ is a field of quotients or fraction field of $Z[\sqrt{3}] = \{a + b\sqrt{3} : a, b \in Z\}$.