Projects and presentations

Presentations during the week of April 2

1. Angle trisection. One of the classic unsolved problems of Greek mathematics was the question of whether angles could be trisected. This was one of the problems shown to be unsolvable in the early nineteenth century. Nevertheless, Archimedes presented a method of angle trisection. What was it, how did it work and why didn't it contradict the methods of Galois?

2. Non-euclidean geometry. This was another subject that arose out of thinking about ancient mathematical problems. Non-euclidean geometry was developed initially to show the independence of the parallel postulate from the other of Euclid's axioms for geometry. What do non-euclidean geometries look like? How are they used in modern times?

3. Goodstein's theorem: Gödel's theorem guarantees that there are sentences true of the natural numbers but not provable say from Peano arithmetic. The sentence that his original proof gave was complicated and somewhat convoluted. There are more "natural" sentences known to be independent of PA; Goodstein's theorem is one of them. What is it, why is it true and for the brave, why is it independent of PA?

4. Strengthened Ramsey's theorem: As with Goodstein's theorem, this is another sentence true of the natural numbers but independent of PA. You should explain what Ramsey's theorem is, and why it and its strengthening are true.

5. Gentzen's theorem: Gödel's theorem definitely didn't settle everything. Gentzen proved a consistency result for a weakened version of Peano arithmetic assuming some basic information about ordinals. What did the theorem say and how is it related to Gödel's result?

6. Non-standard models of Peano arithmetic: If there are sentences that are independent from Peano arithmetic then there must be models or interpretations of arithmetic in which all the axioms of PA are true but in which some sentence true in the natural numbers is in fact false. What do these models look like? How are they built?

7. Hilbert's 10th problem: Hilbert asked for a decision procedure for diophantine equations over the integers. What was the problem and how was it shown to be undecidable? What is its relation to the decidability problems looked at in our course?
8. Decidability of the real and complex fields: Is any part of mathematics decidable? Tarski showed that yes, the theories of the fields of real numbers and the field of complex numbers are decidable. What did he actually prove and how did he prove it?
9. What does Gödel's theorem say about automated theorem proving, machine learning and programme correctness? It clearly puts some limits on what one can do but people go ahead and do things anyway. How does that work and what are the limitations?
10. Anything else you find interesting and is relevant to the course. There are clearly a lot of topics that one could do a project on. Feel free to talk to me if you would like to try something different than what is listed.