Assignment 1, Math 3TP3
Due Jan. 24, in class

1. In class, we showed how Euclid constructed an equilateral triangle. Use this construction to construct a regular hexagon. Generalize this to construct a regular $3 \cdot 2^n$-gon.

2. We called a set scattered if after deriving it sufficiently many times it vanishes. Construct a scattered set A such that you have to derived A $\omega + 1$-many times before the set disappears. Hint: Take the example A we constructed in class of a set such that $A^{(n)}$ was not empty for every n but $\bigcap_{n \in N} A^{(n)} = \emptyset$ and then remember that $\tan^{-1}$ maps $\mathbf{R}$ to the bounded interval $(-\pi/2, \pi/2)$. This allows you to squish even something constructed on all of $\mathbf{R}$ to an interval.

3. Let's create a dictionary between logical notions and characteristic functions: Suppose that A and B are subsets of $\mathbf{N}$. Express, in terms of the characteristic functions χ_A and χ_B, the following:

   (a) the characteristic function of $\mathbf{N} \setminus A$ i.e. the set of elements of $\mathbf{N}$ not in A.

   (b) the characteristic function of $A \cup B$.

   (c) the characteristic function of $A \cap B$.

   (d) Suppose that $A \subset \mathbf{N}^2$. How do you express the characteristic function of "there exists y such that $(x, y) \in A$"?

4. Show that for any set A, there is no surjective map from A to $\mathcal{P}(\mathcal{A})$. This shows that the cardinality of A is less than that of $\mathcal{P}(\mathcal{A})$ for every set A.