Assignment 1, Math 3TP3

Due Jan. 26, deposited in the Avenue dropbox. No photos of the assignment please - proper pdf scans only.

1. In class, we showed how Euclid constructed an equilateral triangle. Use this construction to construct a regular hexagon. Generalize this to construct a regular $3 \cdot 2^n$-gon.

2. Recall that in class we constructed sets A_n for every n such that if one derived them n times they remained non-empty but if you derived them $n+1$ times. This led to the construction of a set A_ω with the property that $A_\omega^{(n)}$ was non-empty for every n but $\bigcap_n A_\omega^{(n)} = \emptyset$. We will now construct a set A such that you have to derived A $\omega + 1$-many times before the set disappears. That is,

$$B = \bigcap_n A^{(n)} \neq \emptyset \text{ but } B' = \emptyset.$$

   Hint: Take the example A_ω we constructed in class as mentioned above. Now recall that $\tan^{-1}$ maps $\mathbf{R}$ to the bounded interval $(-\pi/2, \pi/2)$. This allows you to squish even something constructed on all of $\mathbf{R}$ to an interval. Try to piece together squished copies of A_ω to get the required set A.

3. Let's create a dictionary between logical notions and characteristic functions: Suppose that A and B are subsets of $\mathbf{N}$. Express, in terms of the characteristic functions χ_A and χ_B, the following:

   (a) the characteristic function of $\mathbf{N} \setminus A$ i.e. the set of elements of $\mathbf{N}$ not in A.

   (b) the characteristic function of $A \cup B$.

   (c) the characteristic function of $A \cap B$.

   (d) Suppose that $A \subset \mathbf{N}^2$. How do you express the characteristic function of "there exists y such that $(x, y) \in A$"?

4. Show that for any set A, there is no surjective map from A to $\mathcal{P}(\mathcal{A})$. This shows that the cardinality of A is less than that of $\mathcal{P}(\mathcal{A})$ for every set A.