

Math 3TP3, Midterm Test

Bradd Hart, Feb. 14, 2020

There are 5 questions and each question is worth 5 marks; the test will be graded out of 25.

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. In the language L_A , write formulas which will express the following properties of the natural numbers; don't worry about exact syntax as long as your formula is readable.
 - (a) x is prime.
 - (b) x is divisible by 5.
 - (c) $z = |x - y|$ i.e. a formula in three variables that expresses this relationship.
2. Explain why the following properties are decidable:
 - (a) n is not prime.
 - (b) n is a power of 5.
 - (c) n is even, greater than 2 and the sum of two primes. Bonus: What impact, if any, does knowing that this property is decidable have on the decidability of Goldbach's conjecture?
3. Prove that the power set, the set of all subsets, of the natural numbers is uncountable.
4. Explain why it is possible to effectively enumerate all algorithms.
5. Explain why you think the following properties are decidable or not decidable:
 - (a) The n^{th} decimal digit of π is 2.
 - (b) The sequence 0123456789 appears in the decimal expansion of π .