Basic Definitions and Concepts for Functions

Section 1.1

Relations and Functions

A **relation** between two variables is the set of all ordered pairs of values that occur.

A function is a special type of relation.

Functions

A **function** *f* is a rule that assigns to each element *x* in a set *D* called the domain exactly one element y=*f(x)* in a set *R* called the range.

Functions can be described in 4 ways:

- Numerically (table of values)
- Geometrically (graph)
- Algebraically (explicit formula)
- Verbally (description in words)

Exercise

Given the function $f(x) = x^2 + 2x + 1$
evaluate the following:

"difference quotient"

(a) $f(-1)$

(b) $\dfrac{f(a+h) - f(a)}{h}$

Domain

The **domain of *f*** is the largest set of real numbers (possible x-values) for which the function is defined (as a real number).

Example:

Find the domain of the following functions.

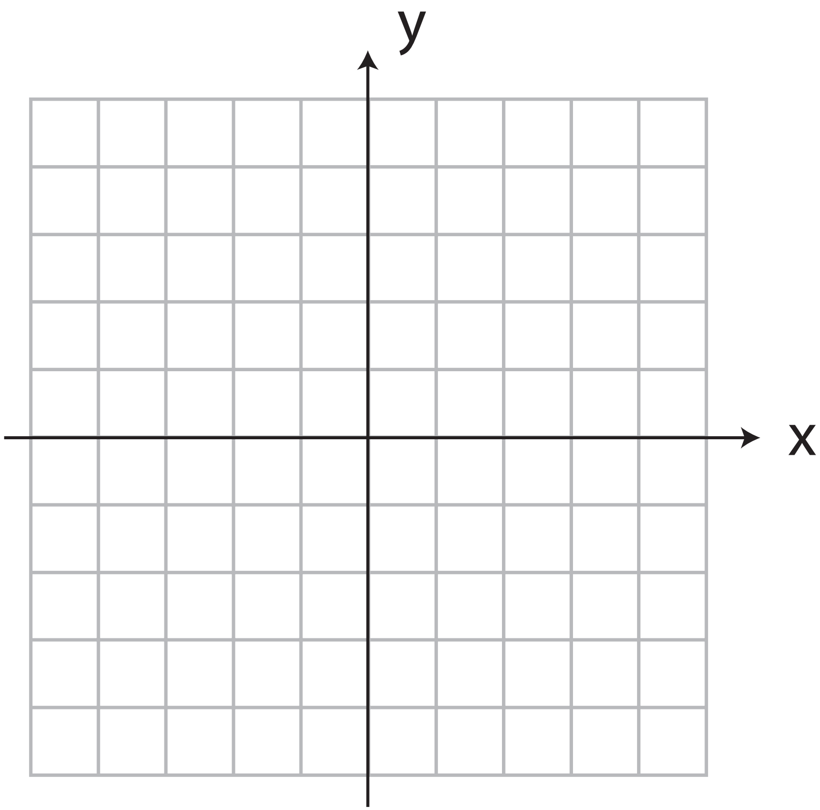
Graphs

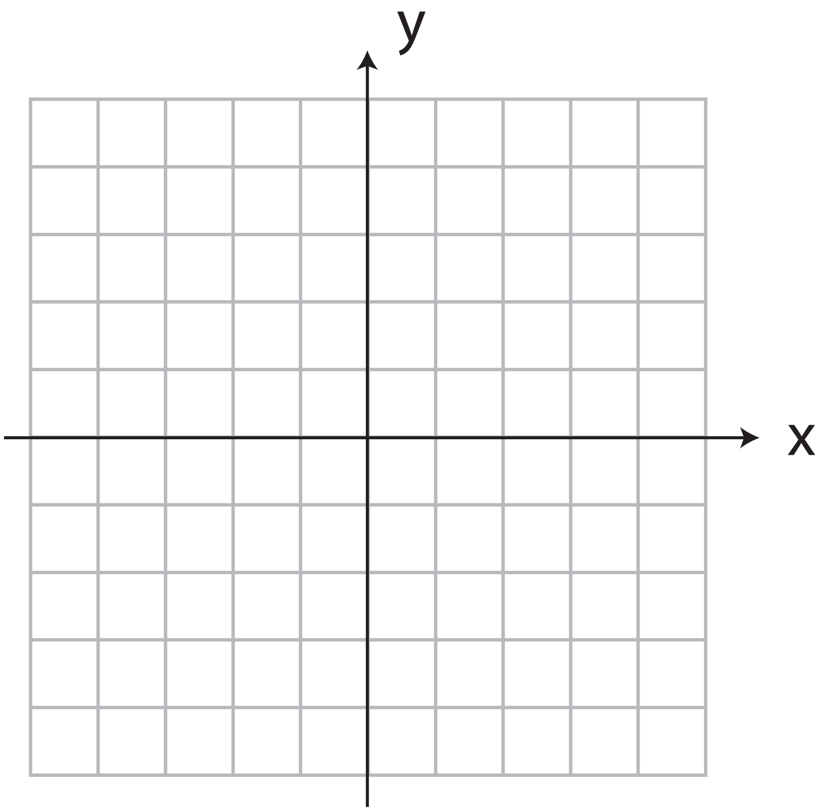
The **graph** of a function *f* is a set of all ordered pairs (points) (x,y) where *x* is in the domain of *f* and $y=f(x)$.

Example:

Sketch the graphs of the following functions. State the domain and range of each.

Graphs





Vertical Line Test

If every vertical line intersects a graph in at most one point, then the graph represents a function.

Examples:

Piecewise Functions

A piecewise function *f(x)* is a function whose definition changes depending on the value of *x*.

Example: *Absolute Value Function*

The **absolute value** of a number *x*, denoted by |*x*|, is the distance between *x* and 0 on the real number line.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

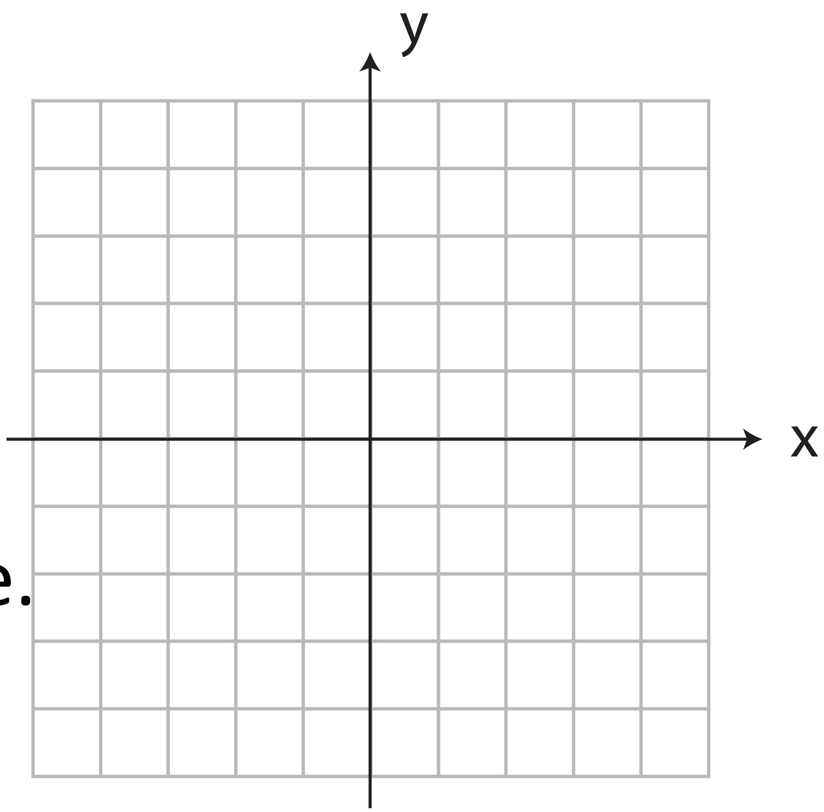
Piecewise Functions

Example:

Sketch the graph of

$$f(x)=\begin{cases} x+4, & x<-1 \\ x^2, & -1\le x\le 2 \\ -2, & x>2 \end{cases}$$

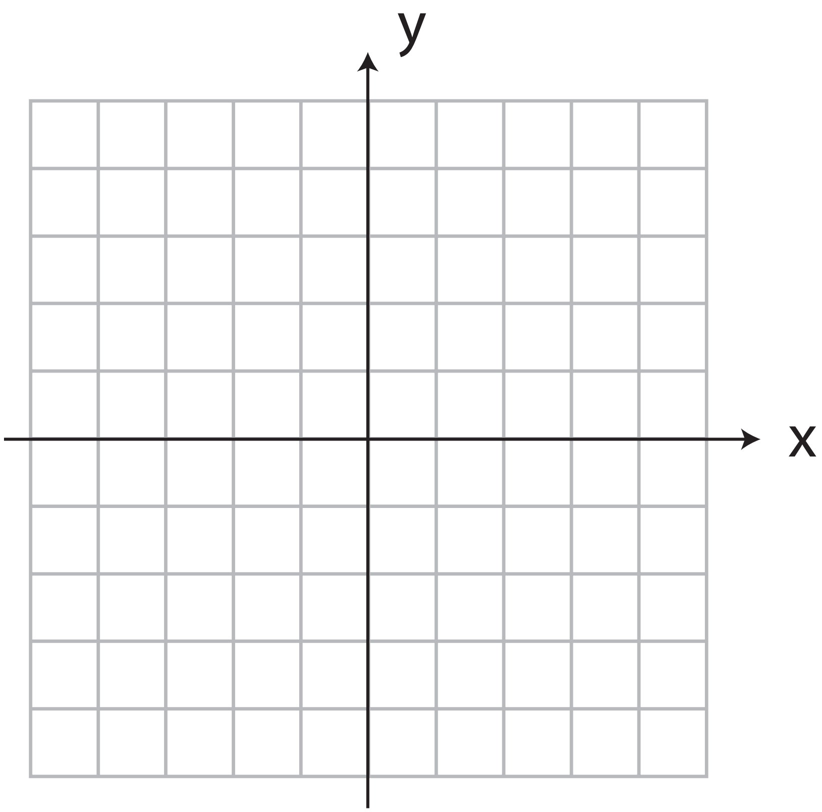
State the domain and range.



Piecewise Functions

Example:
Sketch the graph of

$$f(x) = |1 + 2x|$$



Increasing and Decreasing Functions

A function *f* is called **increasing** on an interval *I* if

$f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in *I*

A function *f* is called **decreasing** on an interval *I* if

$f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in *I*