Vectors

Section 12.2

(Print these slides so you can use the graph paper in them to help with your note taking!)

Mathematical Quantities

Vectors:
quantities that have both magnitude (size) and direction

Examples:

Scalars:
quantities described by magnitude only

Examples:

Vectors

A vector can be represented by a **directed line segment** (arrow).

The **magnitude** of the vector is represented by the **length** of the arrow.

The **direction** of the vector is represented by the direction the arrow is pointing (often given as an **angle**).

Terminology

Equivalent Vectors:
Two vectors are **equivalent** (or equal) if they have the same magnitude and direction (location is irrelevant).

Opposite Vectors:
Two vectors are opposite if the have the same magnitude but point in opposite directions.

Parallel Vectors:
Two vectors are parallel if they point in either the same or opposite directions.

Zero Vector:
Denoted by **0**, the magnitude of the zero vector is 0 and it is the only vector with **no specified direction**.

Combining Vectors

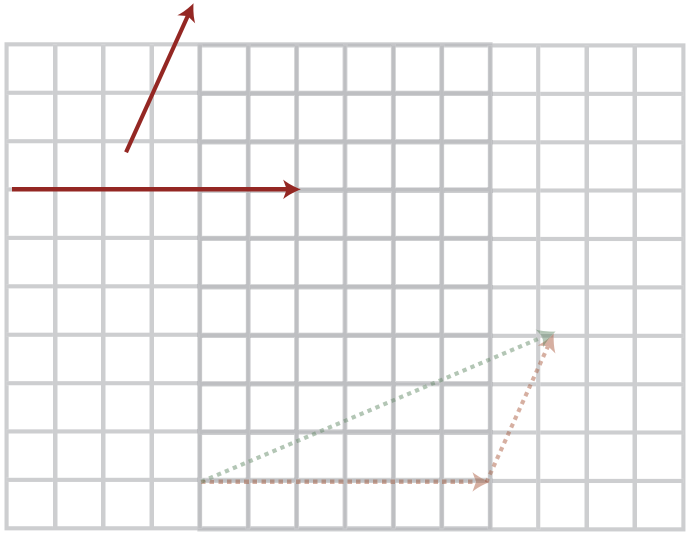
We often want to know the combined effect or <u>sum</u> of two or more vectors.

Example:
How does the wind velocity affect the velocity of a plane?

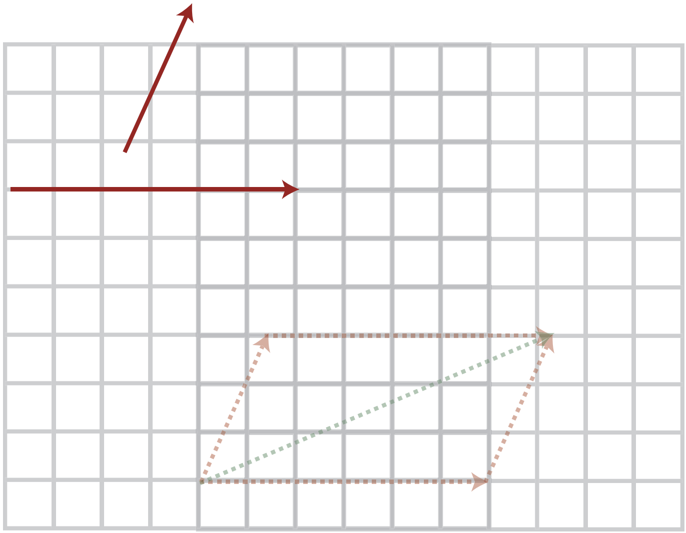
Vector Addition

If **u** and **v** are vectors positioned so that the tail of **v** is at the tip of **u**, then the sum **u+v** is the vector from the tail of **u** to the tip of **v**.

Triangle Law

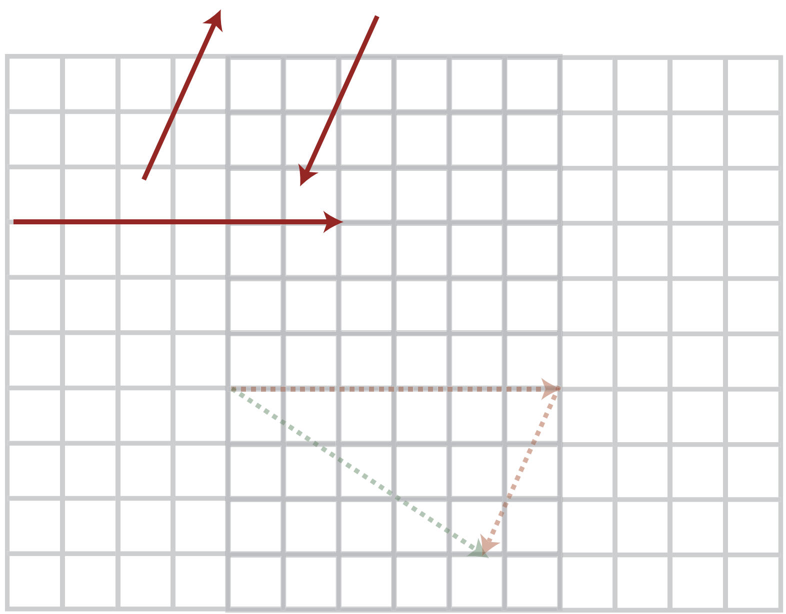


Parallelogram Law



Vector Subtraction

The difference of two vectors, **u**-**v**, is found by adding the vectors **u** and **–v** (the opposite of vector **v**), i.e. **u**-**v**=**u**+(**-v**).

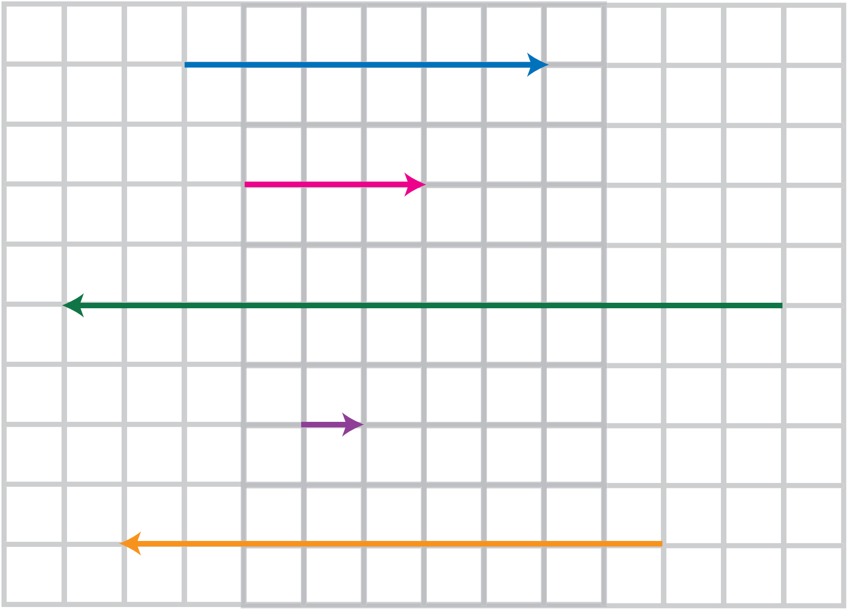


Scalar Multiplication of a Vector

If c is a scalar and **v** is a vector, then the **scalar multiple** c**v** is the vector whose length is |c| times the length of **v**.

The direction of c**v** is the same as **v** if c>0 and is opposite to **v** if c<0.

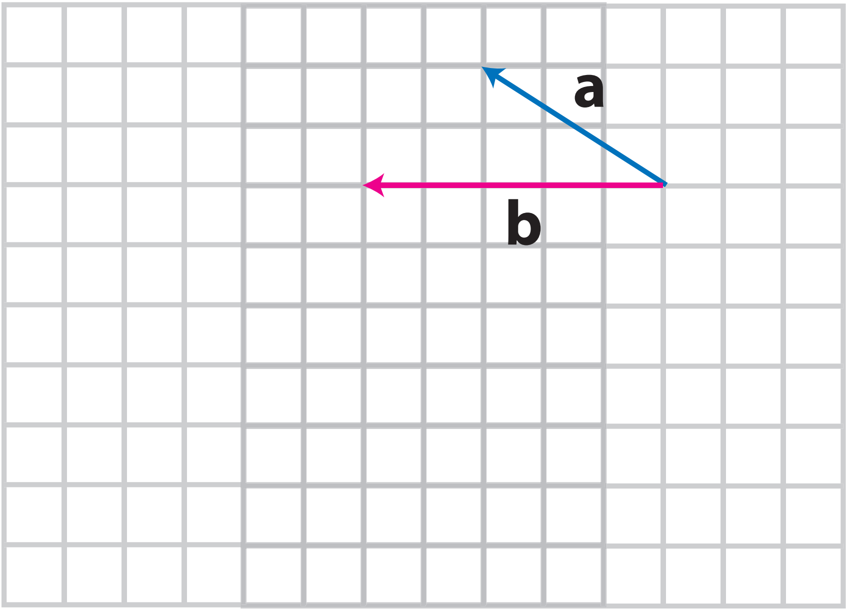
If c=0 or **v**=**0**, then c**v**=**0**.



Combining Vectors

Example:

Use the given vectors to draw the vector 2**b**-**a**.



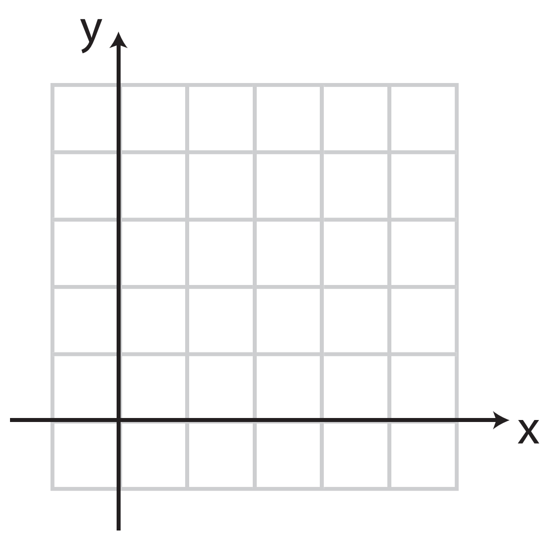
Vectors in a Coordinate System

We can also represent vectors algebraically as an ordered pair **v**=<a,b>.

This is called a **position vector.**

The initial point (tail) is at the origin O(0,0) and its terminal point (head) is at the point P(a,b).

Example:



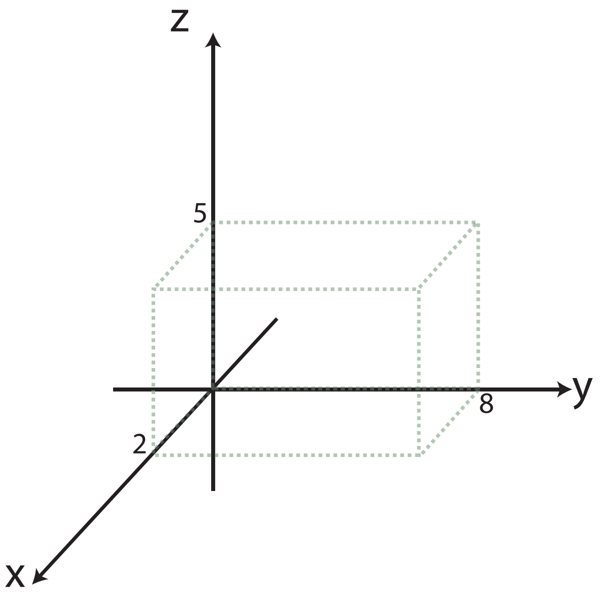
Vectors in a Coordinate System

We represent vectors in 3-dimensions algebraically as an ordered triple **v**=<a,b,c>.

This is called a **position vector.**

The initial point (tail) is at the origin O(0,0,0) and its terminal point (tip or head) is at the point P(a,b,c).

Example:

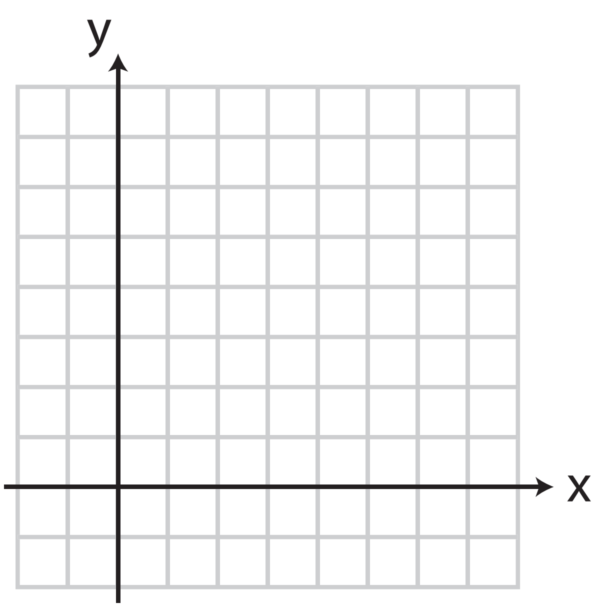


Vectors in a Coordinate System

Given the points $A(x_1, y_1)$ and $B(x_2, y_2)$, the *geometric vector* **AB** is obtained by drawing the vector with its tail at A and its tip at B.

Algebraically, we can find an equivalent position vector **a** by taking **a**$=<x_2 - x_1, y_2 - y_1>$.

Example:



Vectors in a Coordinate System

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the geometric vector **AB** is obtained by drawing the vector with its tail at A and its tip at B.

Algebraically, we can find an equivalent position vector **a** by taking
$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$.

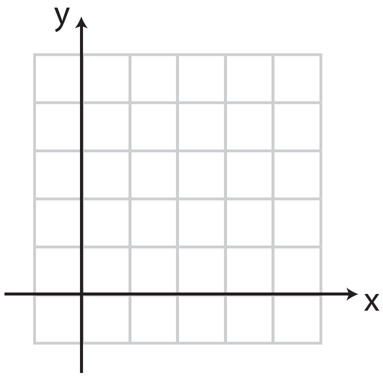
Example:

Magnitude (Length) of Vectors

The length of the two-dimensional vector **a**=<a_1, a_2> is

$$|\mathbf{a}| = \sqrt{a_1^{\,2} + a_2^{\,2}}$$

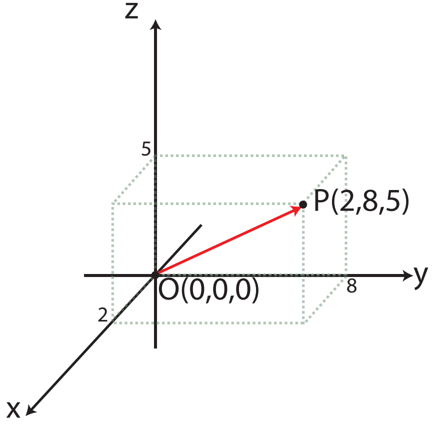
Example:



The length of the three-dimensional vector **a**=<a_1, $a_{2,}$ a_3> is

$$|\mathbf{a}| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$$

Example:



Combining Algebraic Vectors

To add algebraic vectors, we add their components:

If **a**=$\langle a_1, a_2 \rangle$ and **b**=$\langle b_1, b_2 \rangle$, then **a**+**b**=$\langle a_1+b_1 , a_2+b_2 \rangle$

If **a**=$\langle a_1, a_2, a_3 \rangle$ and **b**=$\langle b_1, b_2, b_3 \rangle$, then **a**+**b**=$\langle a_1+b_1 , a_2+b_2 , a_3+b_3 \rangle$

To subtract algebraic vectors, we subtract their components:

If **a**=$\langle a_1, a_2 \rangle$ and **b**=$\langle b_1, b_2 \rangle$, then **a**-**b**=$\langle a_1-b_1 , a_2-b_2 \rangle$

If **a**=$\langle a_1, a_2, a_3 \rangle$ and **b**=$\langle b_1, b_2, b_3 \rangle$, then **a**-**b**=$\langle a_1-b_1 , a_2-b_2 , a_3-b_3 \rangle$

To multiply algebraic vectors by a scalar, we multiply each component by that scalar:

If **a**=$\langle a_1, a_2 \rangle$, then c**a**=$\langle ca_1, ca_2 \rangle$
If **a**=$\langle a_1, a_2 , a_3 \rangle$, then c**a**=$\langle ca_1, ca_2 , ca_3 \rangle$

Combining Algebraic Vectors

Example 1:

If $\mathbf{a}=\langle 0, 3\rangle$ and $\mathbf{b}=\langle 2,1\rangle$,

find $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, and $2\mathbf{b}$.

Example 2:

If $\mathbf{a}=\langle 2, -4, 4\rangle$ and $\mathbf{b}=\langle 0, 2,-1\rangle$,

find $\mathbf{a}+\mathbf{b}$, $2\mathbf{a}-3\mathbf{b}$, and $|\mathbf{a}-\mathbf{b}|$.

Properties of Vectors

(1) $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

(2) $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$

(3) $\mathbf{a} + \mathbf{0} = \mathbf{a}$

(4) $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$

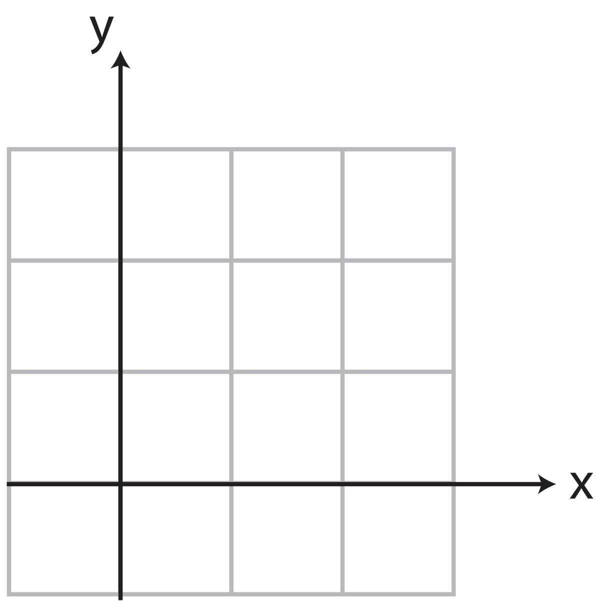
(5) $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$

(6) $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$

(7) $(cd)\mathbf{a} = c(d\mathbf{a})$

(8) $1\mathbf{a} = \mathbf{a}$

Standard Basis Vectors



Any vector in R^2 can be expressed in terms of **i** and **j** as follows:

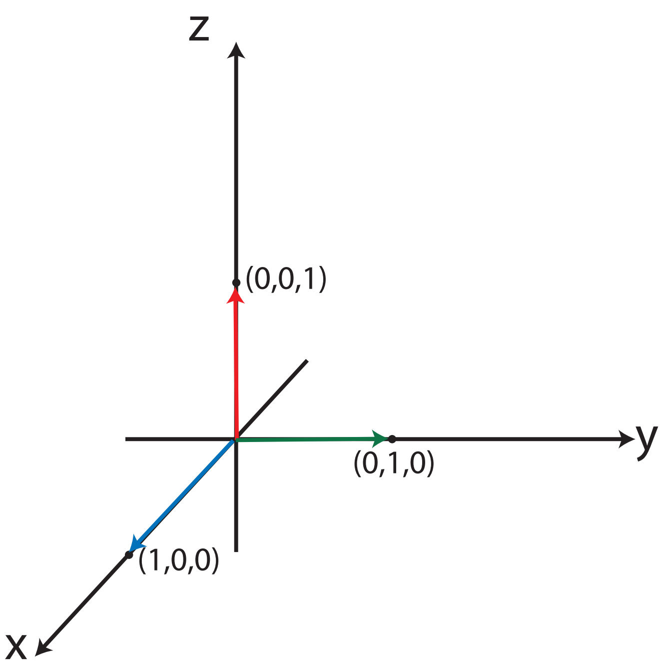
$\mathbf{a} = \langle a_1, a_2 \rangle = a_1\mathbf{i} + a_2\mathbf{j}$

Example:

$\mathbf{i} = \langle 1,0 \rangle$

$\mathbf{j} = \langle 0,1 \rangle$

Standard Basis Vectors



Any vector in R^3 can be expressed in terms of **i**, **j**, and **k** as follows:

$\mathbf{a} = \langle a_1, a_2, a_3 \rangle = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

Example:

$\mathbf{i} = \langle 1,0,0 \rangle$
$\mathbf{j} = \langle 0,1,0 \rangle$
$\mathbf{k} = \langle 0,0,1 \rangle$

Unit Vectors

A **unit vector** is a vector whose length is 1.

Examples:

In general, if $\mathbf{a} \neq \mathbf{0}$, then the unit vector that has the same direction as **a** is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a}$$

Example #24:
Find a unit vector that has the same direction as $-5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.