The Dot Product

Section 12.3

The Dot Product

The **dot product** (or **scalar product**) of two vectors **a** and **b** is the *number* $\mathbf{a} \cdot \mathbf{b}$ defined as follows:

For geometric vectors:

If θ is the angle between the vectors **a** and **b** (defined to be the smaller of the two angles formed when **a** and **b** are placed tail-to-tail), then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

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Example:

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For algebraic vectors in R^2:

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$$

For algebraic vectors in R^3:

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

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Examples:

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Corollary:
If θ is the angle between the nonzero vectors **a** and **b**, then

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$$

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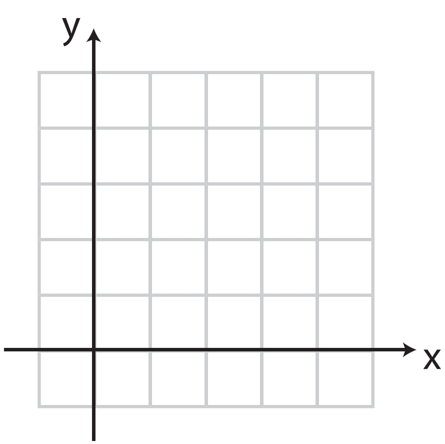
Example:
Find the three angles of the triangle with vertices A(3,0), B(4,4), C(0,2).



Properties of the Dot Product

(1) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

(2) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

(3) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

(4) $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$

(5) $\mathbf{0} \cdot \mathbf{a} = 0$

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Theorem:

Two <u>nonzero</u> vectors **a** and **b** are <u>orthogonal</u> (perpendicular) if and only if $\mathbf{a} \cdot \mathbf{b} = 0.$

Example: