# The Cross Product

Section 12.4

# The Cross Product

The **cross product** (or **vector product**) of two vectors **a** and **b** is the *vector* $\mathbf{a} \times \mathbf{b}$ defined as follows:

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ , then

$$\mathbf{a} \times \mathbf{b} = \left\langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3 , a_1 b_2 - a_2 b_1 \right\rangle$$

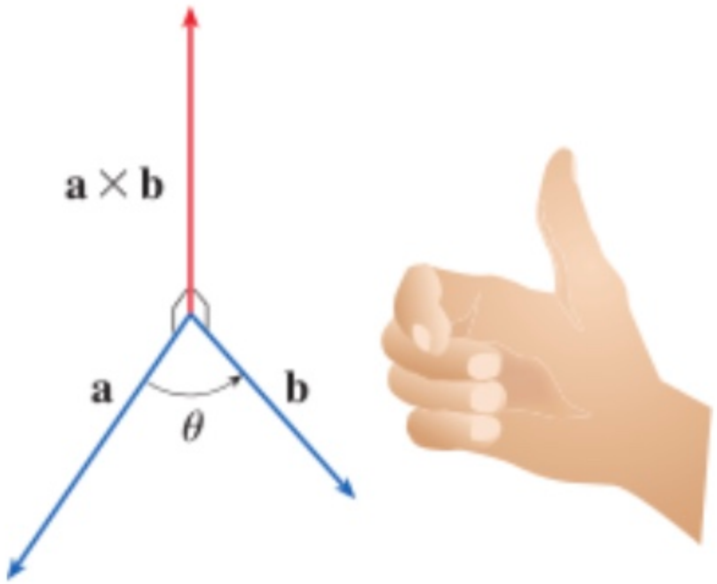
*Note*:
The cross product is only defined in 3D.

# The Cross Product

**Example:**

# The Cross Product

**Theorem:**
The vector $\mathbf{a} \times \mathbf{b}$ is <u>orthogonal</u> (perpendicular) to both **a** and **b**.



# The Cross Product

**Theorem:**

If $\theta$ is the angle between **a** and **b** (so $0 \le \theta \le \pi$), then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

# The Cross Product

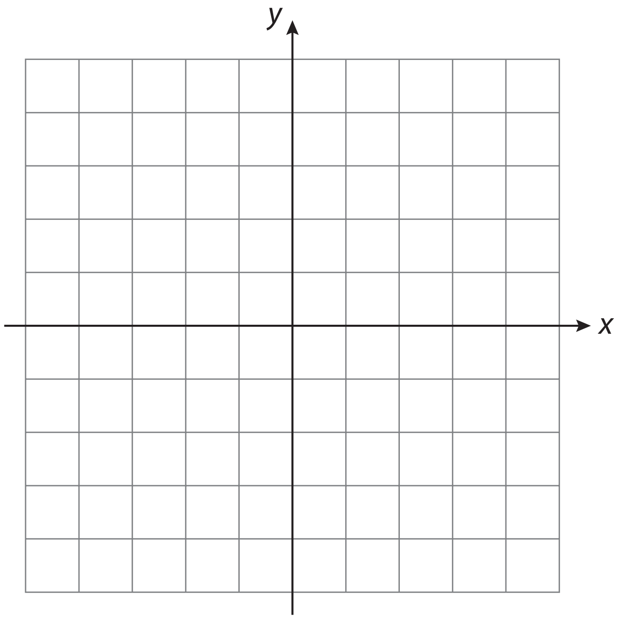
*Application*:

The area of the parallelogram determined by **a** and **b** equals the length of the cross product $\mathbf{a} \times \mathbf{b}$ .

# The Cross Product

**Example:**

Find the area of the parallelogram with vertices A(-3, 0), B(-1, 3), C(5, 2), and D(3, -1).



# Properties of the Cross Product

(1) $\mathbf{a} \times \mathbf{b} = - \mathbf{b} \times \mathbf{a}$

(2) $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$

(3) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

(4) $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$

(5) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(6) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$