

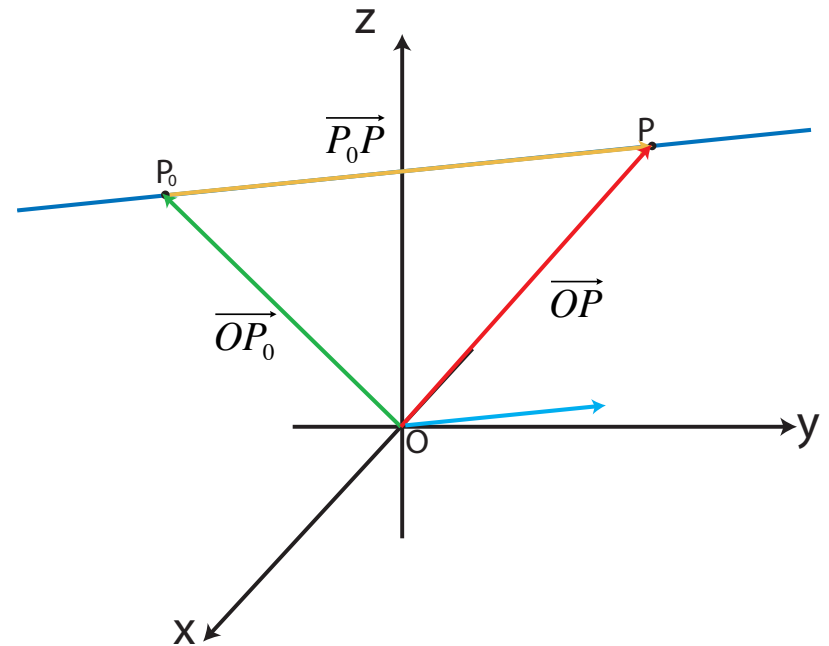
Equations of Lines and Planes

Section 12.5

Vector Equation of a Line in 3D

If $P_0(x_0, y_0, z_0)$ is a known point on the line and $P(x, y, z)$ is an arbitrary point on the line, then by the **triangle law for vector addition**, we can write that

$$\overrightarrow{OP} = \overrightarrow{OP_0} + \overrightarrow{P_0P}$$



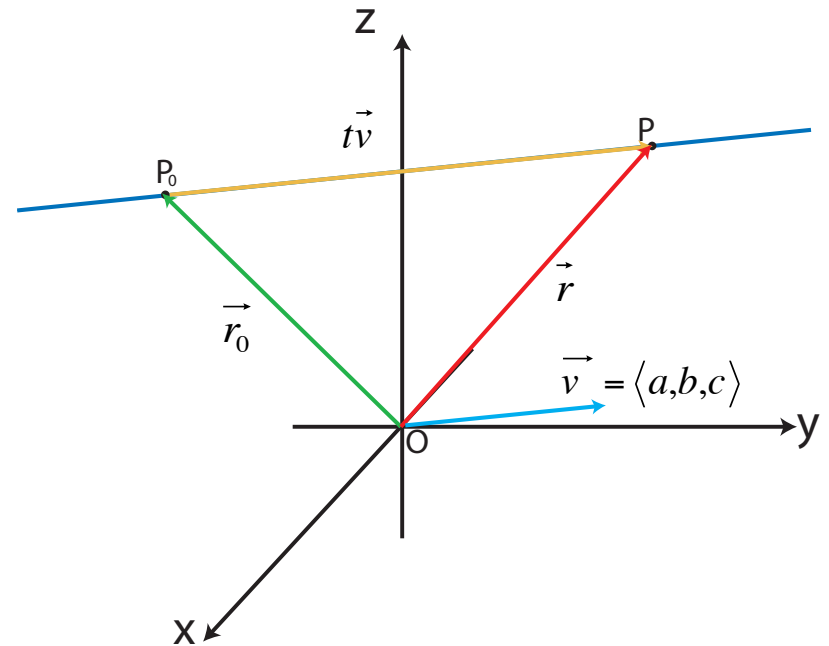
Vector Equation of a Line in 3D

Let vector $\mathbf{v}=\langle a,b,c\rangle$ be any vector parallel to the line (often called a “**direction vector**” for the line).

If we let \mathbf{r} be the the position vector \mathbf{OP} and \mathbf{r}_0 be the position vector \mathbf{OP}_0 , then the **vector equation** of the line can be written as

$$\vec{r} = \vec{r}_0 + t\vec{v}$$

where t is called a parameter and can be any real number.



Parametric Equations of a Line in 3D

If we express the vectors algebraically, we have that

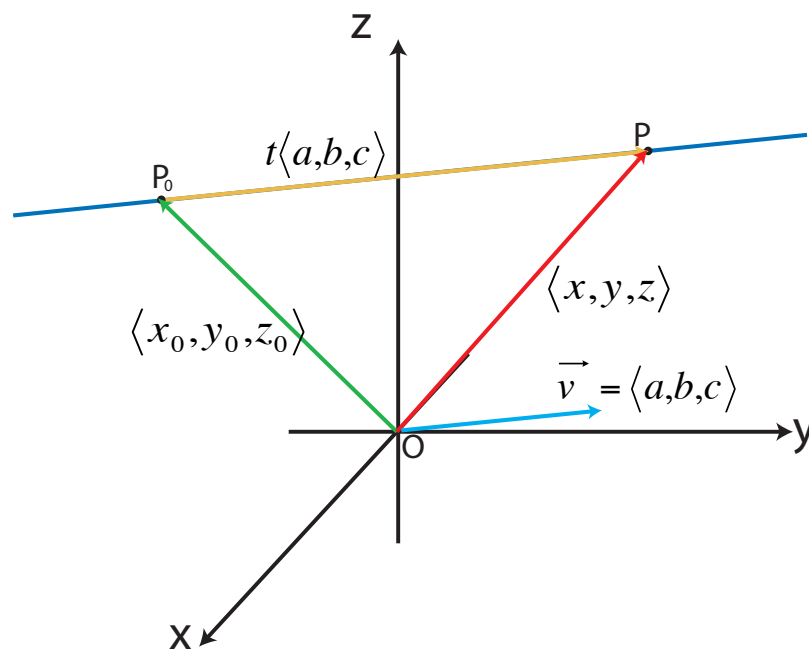
$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

Separating the x , y , and z components, we obtain the **parametric equations** of the line:

$$x = x_0 + at$$

$$y = y_0 + bt$$

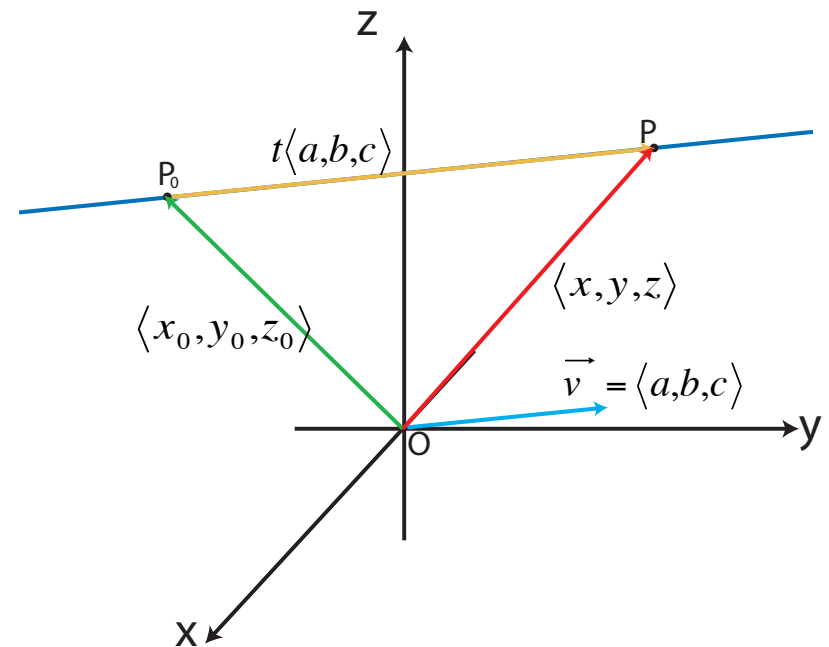
$$z = z_0 + ct$$



Symmetric Equations of a Line in 3D

If we then solve these equations for t (provided that the **direction numbers**, a , b , and c are not zero) and equate the components, we obtain the **symmetric equations** for the line:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$



Equations of Lines in 3D

Example 1:

Find vector, parametric, and symmetric equations of the line through $P(1,0,3)$ and parallel to the line $\vec{r} = \langle 1, -6, 0 \rangle + t\langle 5, 7, -1 \rangle$.

Equations of Lines in 3D

Example 2:

(a) Find vector, parametric, and symmetric equations of the line through points $P(1,2,0)$ and $Q(-3,2,1)$.

(b) At which point does this line intersect the xy -plane?

Equation of a Plane

A plane is determined by a point $P_0(x_0, y_0, z_0)$ in the plane and a vector \mathbf{n} , called a normal vector, that is perpendicular to the plane.

Equation of a Plane

Let $\mathbf{n}=\langle a,b,c\rangle$ be a normal vector for the plane. This normal vector, or simply normal, is perpendicular to every vector in the plane.

If $P_0(x_0, y_0, z_0)$ is a known point in the plane and $P(x,y,z)$ is an arbitrary point in the plane, then \mathbf{n} is perpendicular to $\mathbf{P}_0\mathbf{P}$ and so we have that

$$\vec{n} \cdot \overrightarrow{P_0P} = 0$$

Equation of a Plane

Using the definition of the dot product for algebraic vectors, we obtain the **scalar equation of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $n = \langle a, b, c \rangle$:**

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

This equation is often expanded and simplified to

$$ax + by + cz + d = 0$$

where $d = -(ax_0 + by_0 + cz_0)$.

Equations of Planes

Example #26.

Find an equation of the plane through the point $(2,0,1)$ and perpendicular to the line $x=3t$, $y=2-t$, $z=3+4t$.

Equations of Planes

Example #32.

Find an equation of the plane through the origin and the points $(3,-2,1)$ and $(1,1,1)$.

Equations of Planes

Example #34.

Find an equation of the plane that passes through the point $(1,2,3)$ and contains the line $x=3t$, $y=1+t$, $z=2-t$.