Derivatives and Rates of Change

Section 2.7 (+ a bit from 2.1)

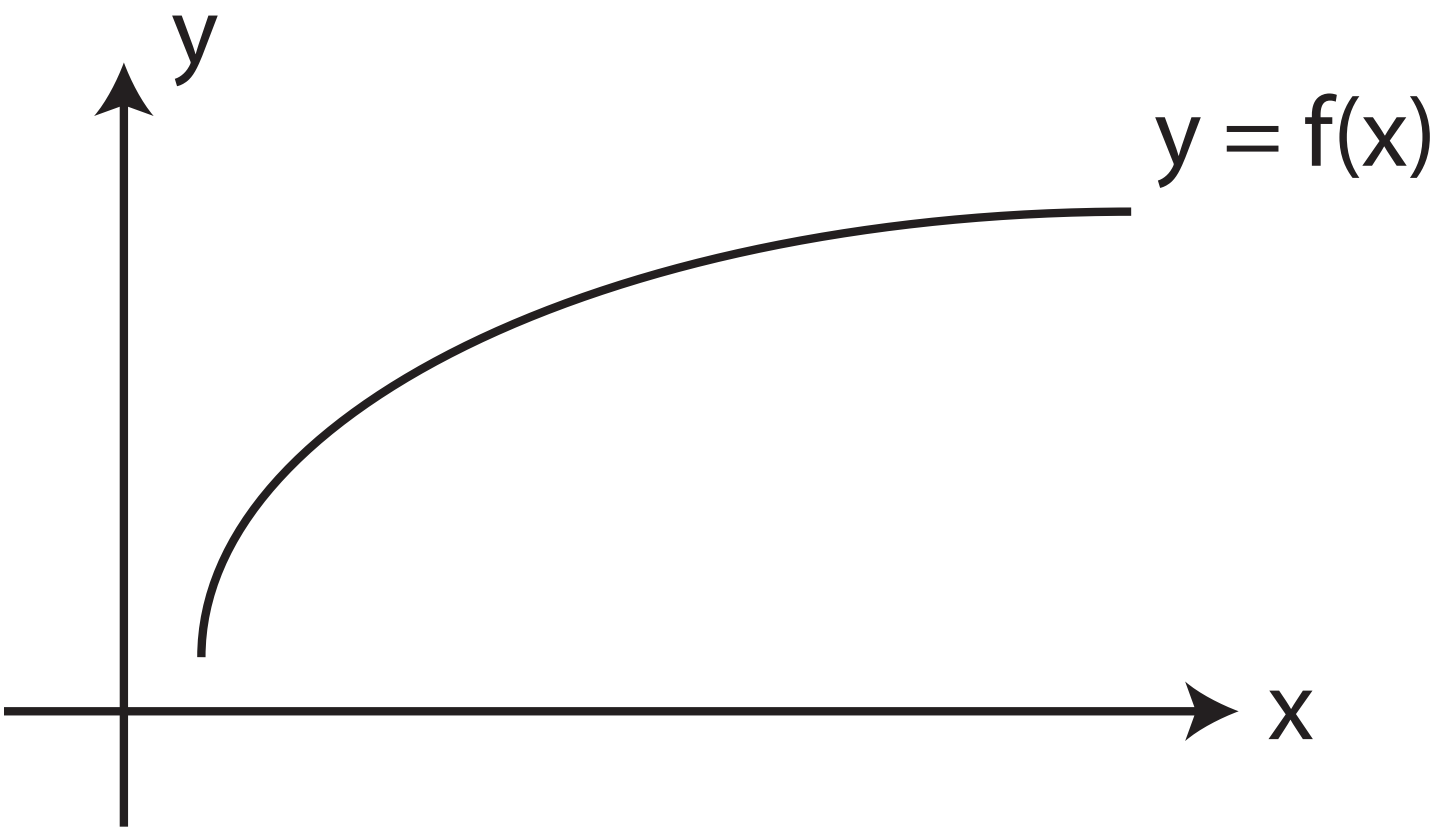
Rates of Change

The rate of change of a function tells us how the dependent variable changes when there is a change in the independent variable.

Geometrically, the **rate of change** of a function corresponds to the **slope** of its graph.

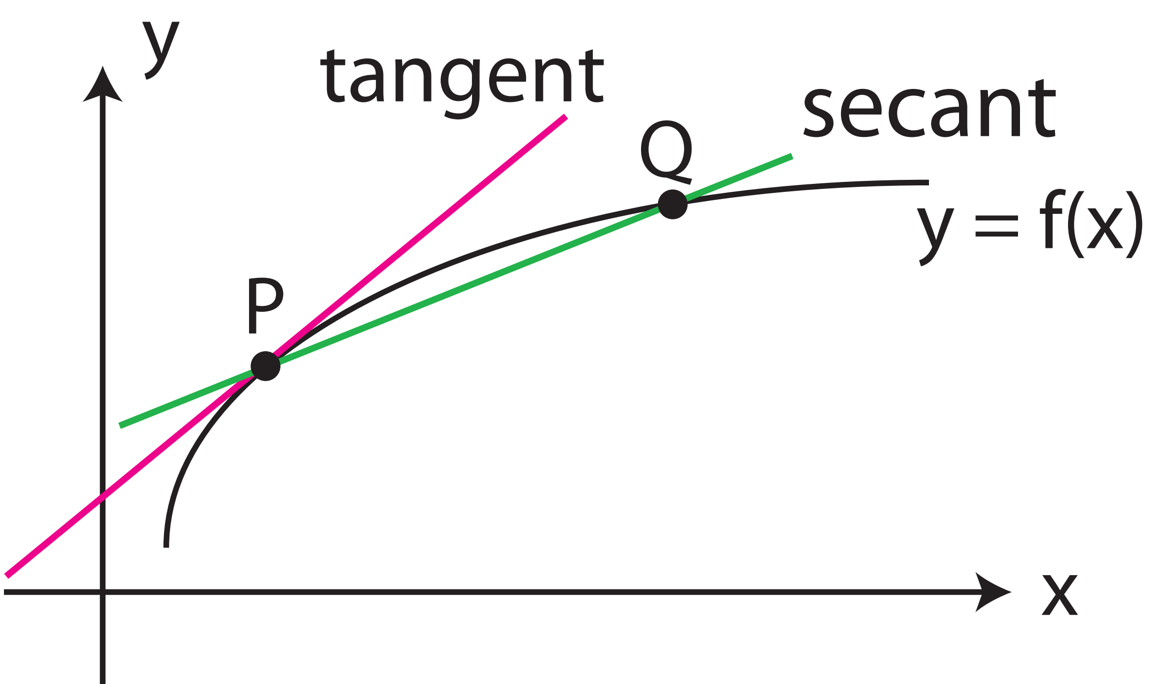
Slope of a Curve

How do we calculate the slope of a curve?



Secant Lines and Tangent Lines

A secant line is a line that intersects two points on a curve.
A tangent line is a line that just touches a curve at a point and most closely resembles the curve at that point.

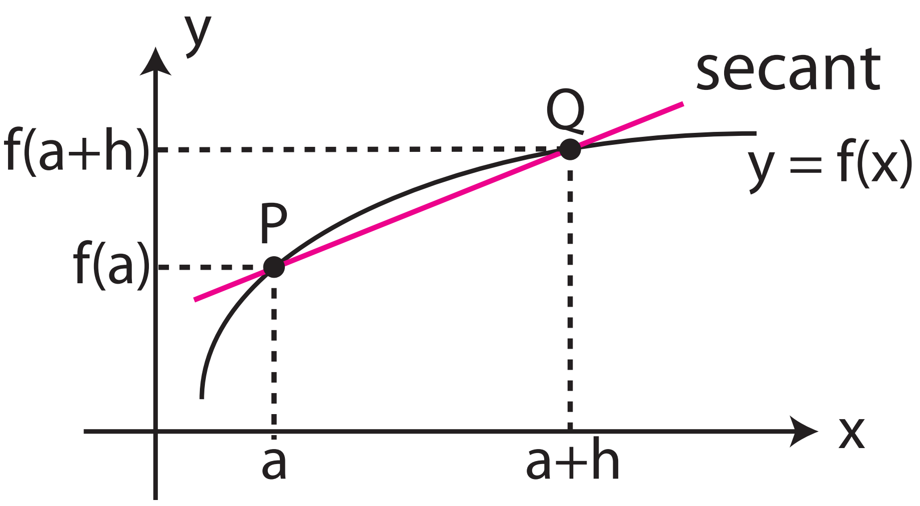


Average Rate of Change = Slope of Secant Line

The average rate of change in *f* from *x=a* to *x=a+h* corresponds to the slope of the secant line *PQ*.

$$m_{PQ} = \frac{f(a+h) - f(a)}{a+h-a}$$

$$= \frac{f(a+h) - f(a)}{h}$$



Estimating the Slope of the Tangent

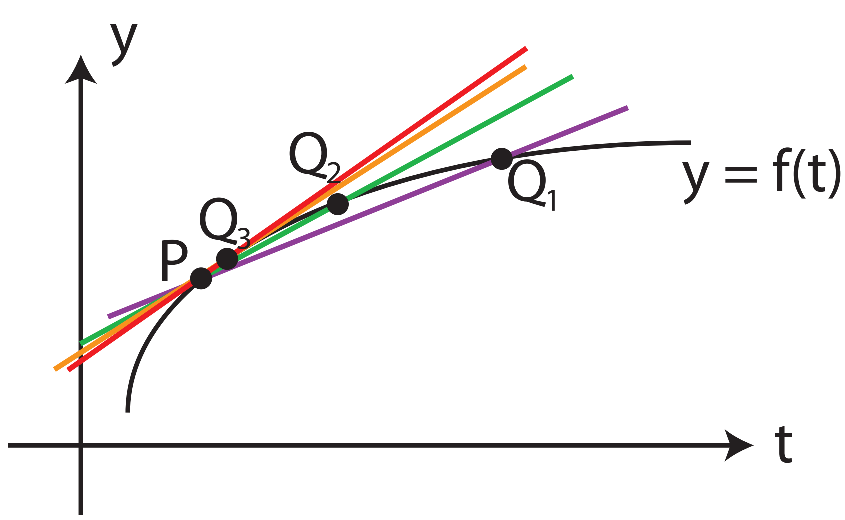
Steps:

1. Approximate the tangent at *P* using secants intersecting *P* and a nearby point *Q*.

2. Obtain a better approximation to the tangent at *P* by moving *Q* closer to *P*, but $Q \neq P$.

3. Define the slope of the tangent at *P* to be the limit of the slopes of secants *PQ* as *Q* approaches *P* (if the limit exists):

$$m_P = \lim_{Q \to P} m_{PQ}$$



Estimate the slope of the tangent to $f(x) = \ln x$ at the base point $P(1,0)$.

Approach 1 from the left:

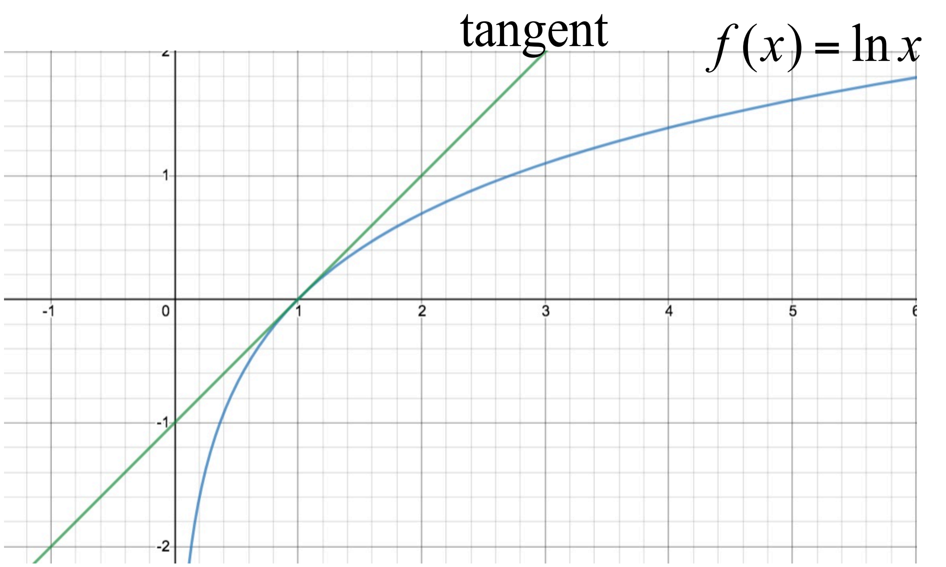
Q	x	y = f(x)	m_{PQ}
Q_1	0.5		
Q_2	0.9		
Q_3	0.99		

Note: This is an overestimate.

Approach 1 from the right:

Q	x	y = f(x)	m_{PQ}
Q_4	1.5		
Q_5	1.1		
Q_6	1.01		

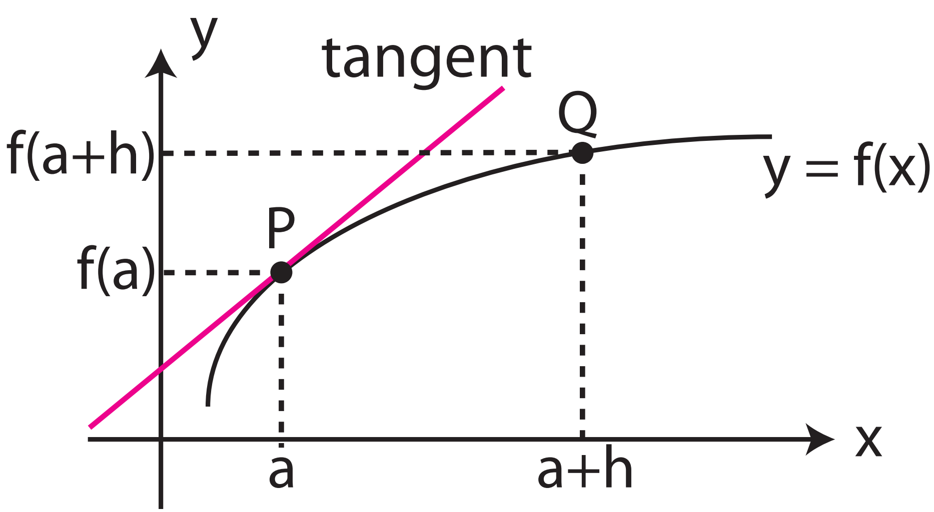
Note: This is an underestimate.



Instantaneous Rate of Change = Slope of Tangent Line

The instantaneous rate of change of *f(x)* at *x=a* corresponds to the slope of the tangent line at *x=a*.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



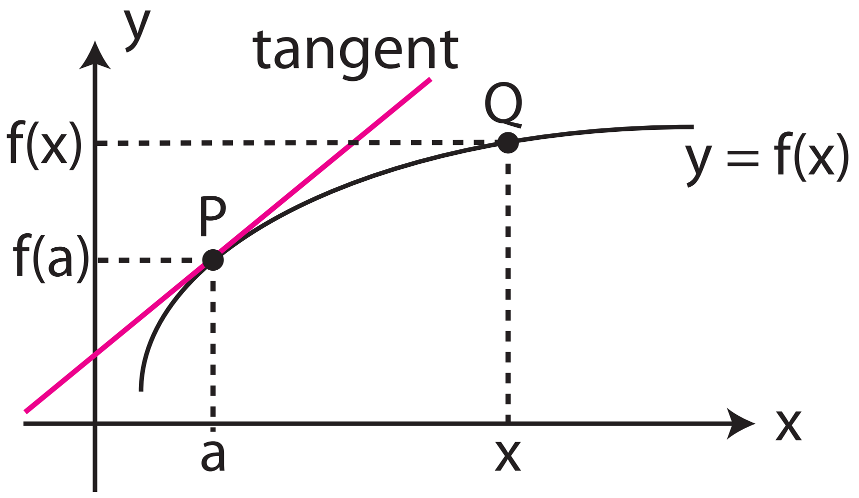
Note:
The slope of the curve *y=f(x)* at *P* is the slope of its tangent line at *P*.

Instantaneous Rate of Change = Slope of Tangent Line

This special limit is called the ***derivative of f at a*** and is denoted by f’(a) (read “f prime of a”).

Alternative formula:

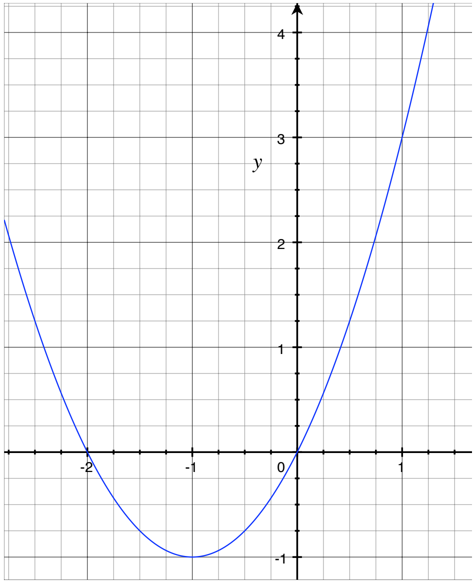
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



Instantaneous Rate of Change = Slope of Tangent Line

Example:

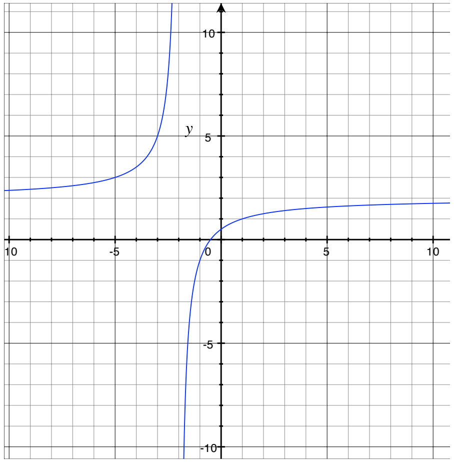
Determine an equation of the tangent line to the curve $f(x) = x^2 + 2x$ at P(1,3).



Instantaneous Rate of Change = Slope of Tangent Line

Example:

#8. Find an equation of the tangent line to the curve $f(x) = \dfrac{2x+1}{x+2}$ at the point (1,1).



The Derivative as a Number

Summary:

The derivative of f at x=a, denoted by **f'(a)**, is a **number** given by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that this limit exists.

The Derivative as a Number

Summary:

* Geometrically, the number f'(a) represents the **slope of the tangent** to the graph of f(x) at the point (a, f(a)).

* The number f'(a) also represents the **instantanous rate of change** of the function f(x) with respect to x at the exact moment when x=a.