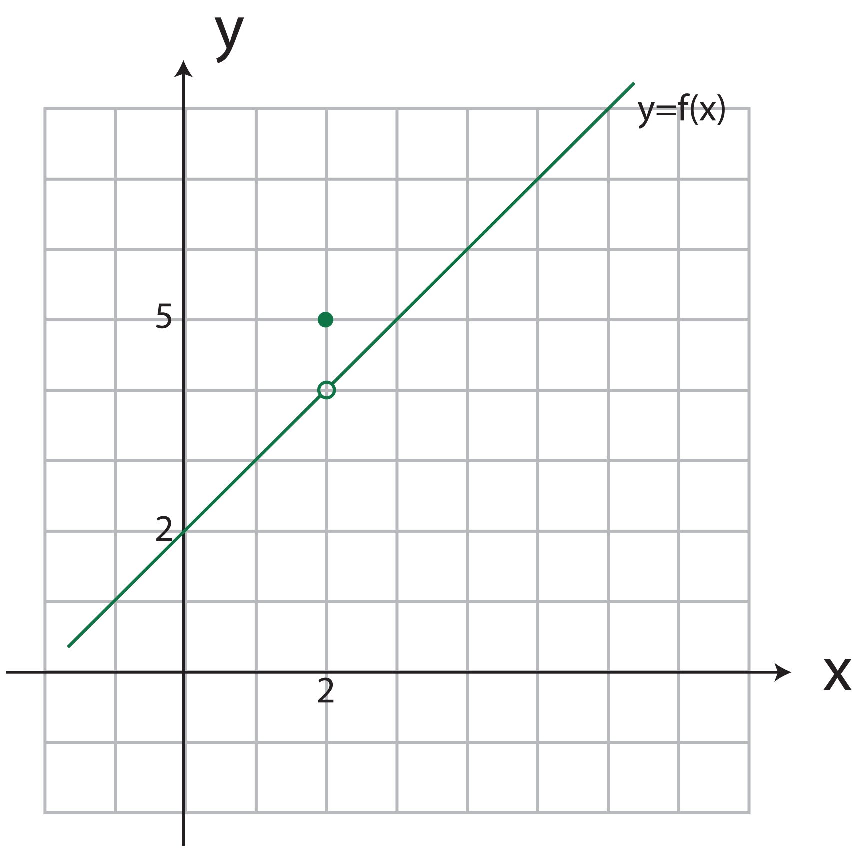
The Limit of a Function

Section 2.2

The Limit of a Function



$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$

Notations:

$$f(2) = 5$$

means that the y-value of the function AT x=2 equals 5

$$\lim_{x \to 2} f(x) = 4$$

means that the y-values of the function APPROACH 4 as x APPROACHES 2

Intuitive Definition of a Limit

Suppose *f(x)* is defined when *x* is near the number *a*. Then we write

$$\lim_{x \to a} f(x) = L$$

and say

"the limit of *f*(*x*), as *x* approaches *a*, equals *L*"

if we can make the values of *f(x)* arbitrarily close to *L* by restricting *x* to be sufficiently close to *a* (on either side of *a*) but not equal to *a*.

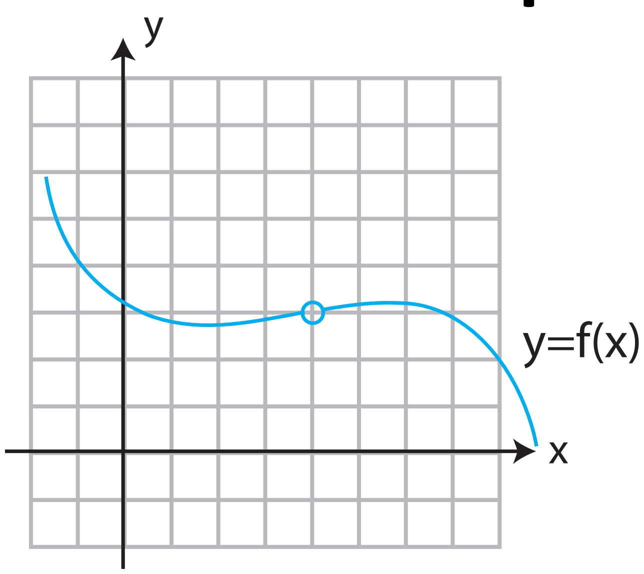
Evaluating Limits

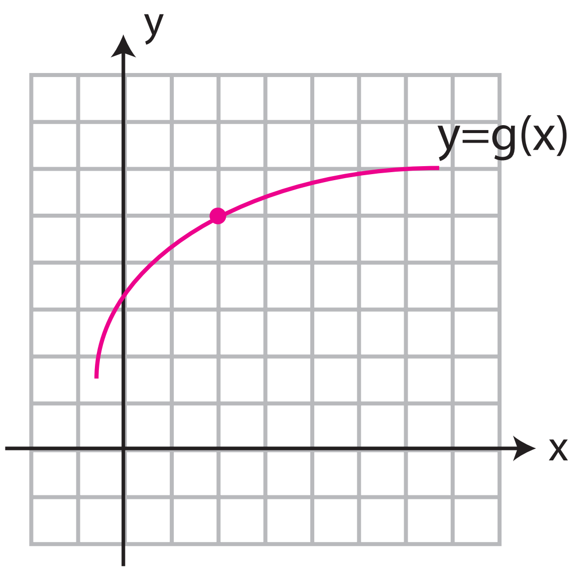
We can evaluate the limit of a function in 3 ways:

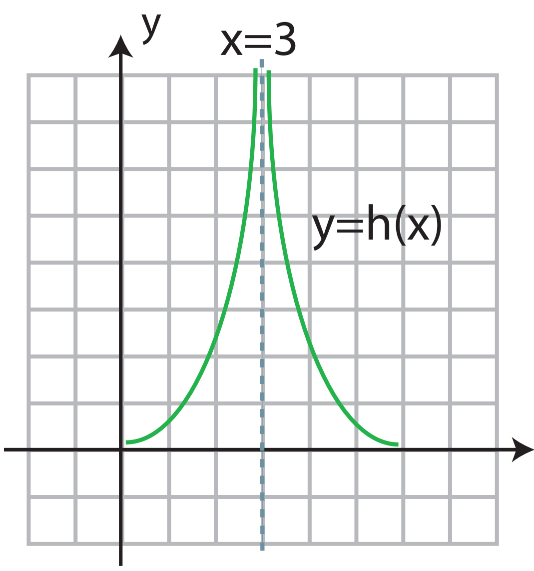
1. **Graphically**
2. **Numerically**
3. Algebraically

Evaluating Limits Graphically

Some examples:







Note: f may or may not be defined at x=a. Limits are only asking how f is defined **NEAR** a.

Left-Hand and Right-Hand Limits

$$\lim_{x \to a^-} f(x) = L$$

means $f(x) \to L$ as $x \to a$ from the left $(x < a)$.

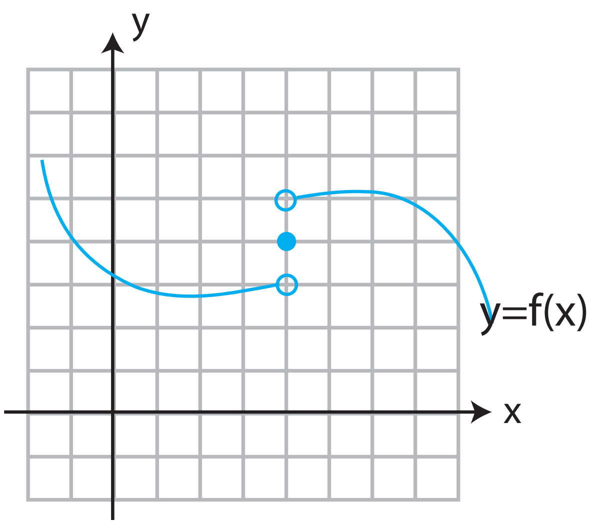
$$\lim_{x \to a^+} f(x) = L$$

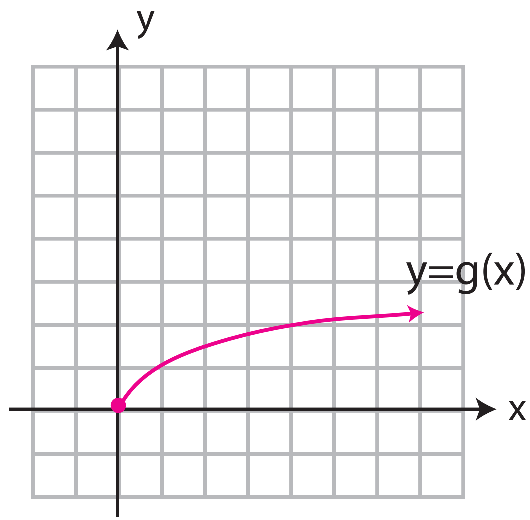
means $f(x) \to L$ as $x \to a$ from the right $(x > a)$.

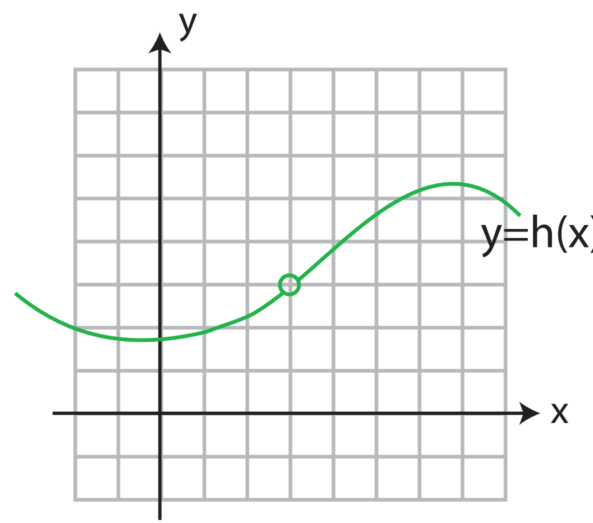
** The full limit exists if and only if the left and right limits both exist (equal a real number) and are the same value.

Left-Hand and Right-Hand Limits

For each function below, determine the value of the limit or state that it does not exist.







Evaluating Limits Numerically

Example:
Use a table of values to estimate the value of

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

x	f(x)
3.5	
3.9	
3.99	
4	undefined
4.01	
4.1	
4.5	

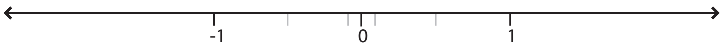
Evaluating Limits Numerically

Example:

Use a table of values to estimate the value of

$$\lim_{x \to 0} \frac{1}{x}$$

x	f(x)
0.1	
0.01	
0.001	
0	undefined
-0.001	
-0.01	
-0.1	



Intuitive Definition of an Infinite Limit

Let *f* be a function defined on both sides of *a*, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of *f(x)* can be made arbitrarily large (as large as we'd like) by taking *x* sufficiently close to *a*, but not equal to *a*.

Intuitive Definition of an Infinite Limit

Let *f* be a function defined on both sides of *a*, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of *f(x)* can be made arbitrarily large negative by taking *x* sufficiently close to *a*, but not equal to *a*.

Vertical Asymptotes

<u>Definition</u>:

The line *x*=*a* is called a **vertical asymptote** of the curve *y*=*f*(*x*) if either

$$\lim_{x \to a-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a+} f(x) = \pm\infty$$

Vertical Asymptotes

Examples:

Find the vertical asymptote(s) of each function.

(a) $f(x) = \dfrac{x+2}{x+3}$

(b) $g(x) = \cot x$