Calculating Limits Using the Limit Laws

Section 2.3

LIMIT LAWS
[used to evaluate limits algebraically]

Suppose that *c* is a constant and the limits

$$\lim_{x \to a} f(x) \text{ and } \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} \left[c f(x)\right] = c \lim_{x \to a} f(x)$

LIMIT LAWS
[used to evaluate limits algebraically]

Continued…

4. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} [f(x) \div g(x)] = \lim_{x \to a} f(x) \div \lim_{x \to a} g(x), \text{ if } \lim_{x \to a} g(x) \neq 0$

6. $\lim_{x \to a} \left[f(x)\right]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer.

7. $\lim_{x \to a} c = c$

8. $\lim_{x \to a} x = a$

LIMIT LAWS
[used to evaluate limits algebraically]

Continued…

9. $\lim_{x \to a} x^n = a^n$ where n is a positive integer.

10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer.

(if n is even, we assume that $a > 0$.)

11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer.

(if n is even, we assume that $\lim_{x \to a} f(x) > 0$.)

Evaluating Limits Algebraically

Example:

Evaluate the limit and justify each step by indicating the appropriate Limit Laws.

$$\lim_{x \to 1} (x^2 + 5x - 6)$$

$$= \lim_{x \to 1} x^2 + \lim_{x \to 1} 5x - \lim_{x \to 1} 6$$

$$= \left[\lim_{x \to 1} x\right]^2 + 5\lim_{x \to 1} x - \lim_{x \to 1} 6$$

$$= 1^2 + 5(1) - 6$$

$$= 0$$

Evaluating Limits Algebraically

From the previous slide, we have

$$\lim_{x\to 1}(x^2+5x-6)=0$$

$f(x)$ $\quad$ $f(1)$

Notice that we could have simply found the value of the limit by plugging in x=1 into the function.

Direct Substitution Property

Direct Substitution Property:
If f is a polynomial or rational function, and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

Example:

Calculate $\lim_{x \to 0} \sqrt[3]{\dfrac{x^2+16}{x-2}}$

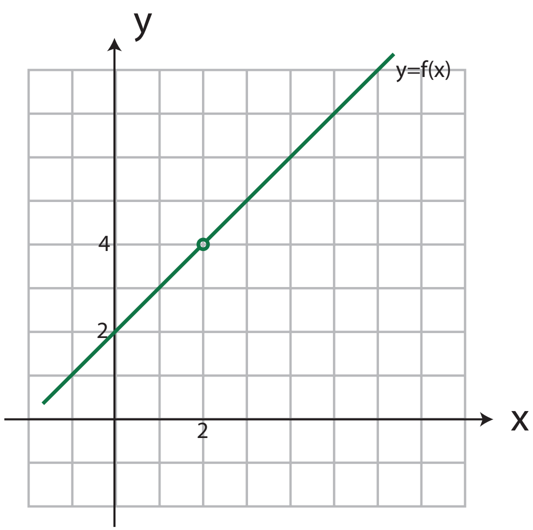
Equal Limits Property

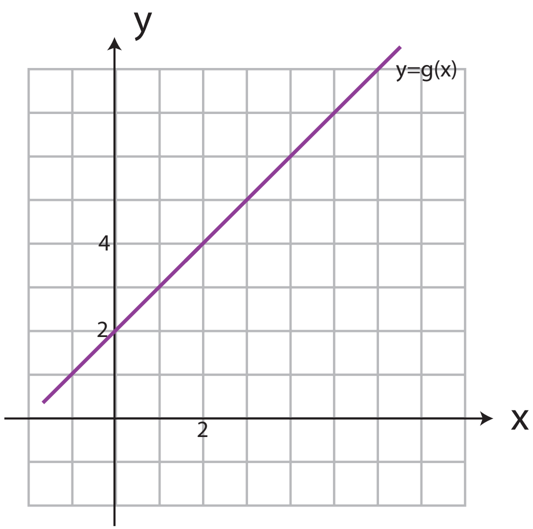
If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$ provided the limits exist.

Equal Limits Property

Consider the functions:

$$f(x) = \frac{x^2 - 4}{x - 2} \qquad\qquad g(x) = x + 2.$$





* *__Note__*: f(x)=g(x) everywhere except at x=2

Equal Limits Property

Example:

Calculate $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$.

Note: direct substitution does not work

Exercises

Evaluate each limit or state that it does not exist.

(a) $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

(b) $\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$

(c) $\lim_{x \to -6} \frac{2x + 12}{|x + 6|}$