Continuity

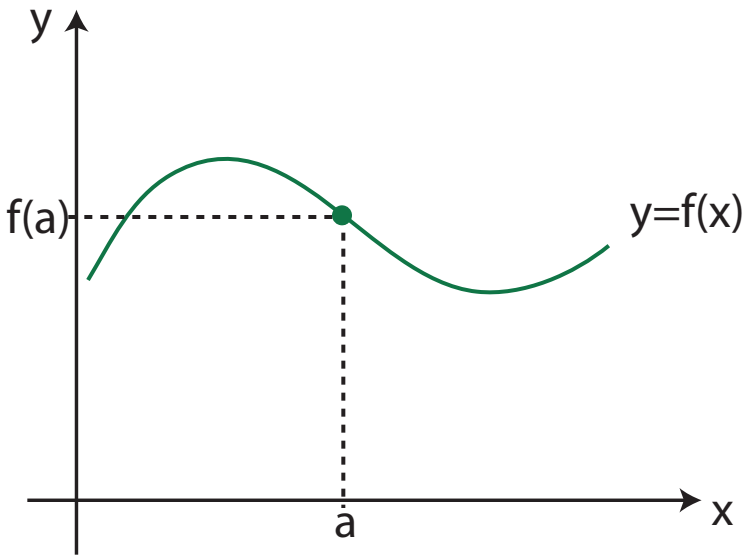
Section 2.5

Continuity

Definition:

A function *f* is **continuous at the point** $x=a$ if $f(x)$ approaches $f(a)$ as x approaches a, i.e.

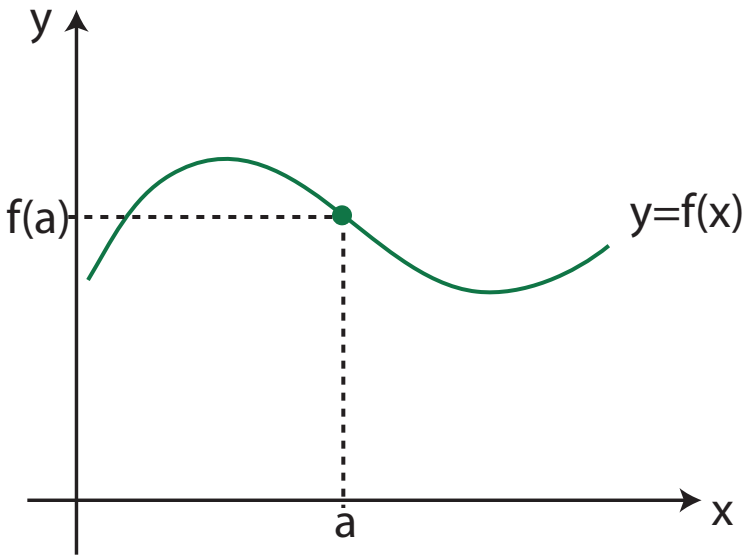
$$\lim_{x \to a} f(x) = f(a)$$



Continuity

Implicitly requires 3 things:

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ is defined
3. $\lim_{x \to a} f(x) = f(a)$



If *f* is *not* continuous at *a* (i.e. *f* fails to meet at least one of the three conditions above), then we say that *f* is **discontinuous** at *x*=*a*.

Exercise

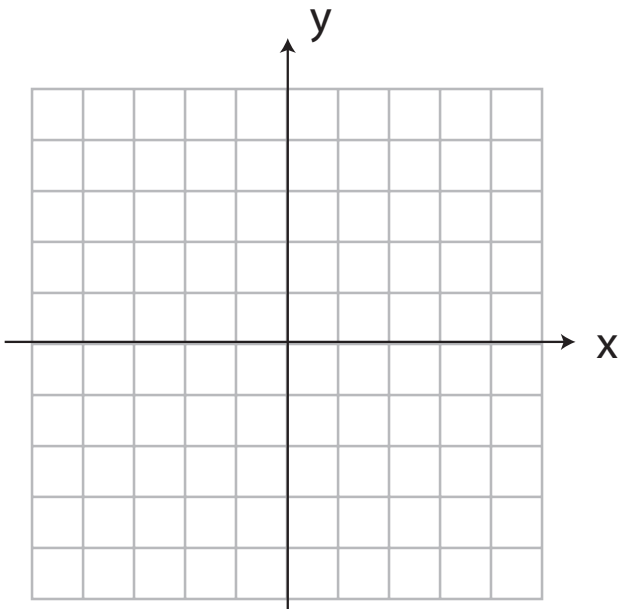
Determine the value of *k* such that *g(x)* is continuous at *x=3*.

$$g(x) = \begin{cases} x + 3 & x \neq 3 \\ 2 + \sqrt{k} & x = 3 \end{cases}$$

Continuity

Example:
Find the discontinuities of the function and explain why it is discontinuous there.

$$h(x) = \frac{x+1}{x^2 - 2x - 3}$$

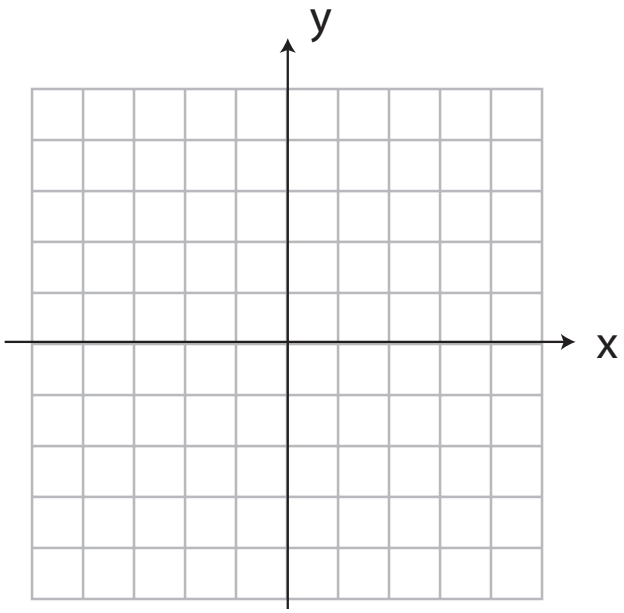


(Start by looking at x-values where f(x) is **not defined** and then check the 3 conditions of continuity.)

Continuity

Example:
Find the discontinuities of the function and explain why it is discontinuous there.

$$g(x) = \begin{cases} 2 - x & x \le 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$



(Start by looking at x-values where f(x) is **changes** from one 'piece' to another and then check the 3 conditions of continuity.)

Which Functions Are Continuous?

A function is **continuous on an interval** if it is continuous at every number in that interval.

The following types of functions are continuous at every number in their domains:

- ✓ Polynomials
- ✓ Rational Functions
- ✓ Root Functions
- ✓ Trigonometric Functions
- ✓ Inverse Trigonometric Functions
- ✓ Exponential Functions
- ✓ Logarithmic Functions

Which Functions Are Continuous?

Combining Continuous Functions:

The sum, difference, product, quotient, and composition of continuous functions is continuous where defined.

Example:

Determine where $g(x) = \sqrt{\ln x - 1}$ is continuous.

Limits of Continuous Functions

By the definition of continuity, if a function is continuous at $x=a$, then we can evaluate the limit simply by direct substitution.

Example:

Evaluate $\lim_{x \to 3} \sqrt{\ln x - 1}.$

Interchanging a Limit and a Continuous Function

Theorem:

If *f* is continuous at *b* and $\lim_{x \to a} g(x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$.

In other words,

$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x))$$

Example:

Evaluate $\lim_{x \to \infty} e^{1+3x-x^2}$.