The Derivative as a Function

Section 2.8

The Derivative as a Function

Definition:

Given a function *f(x)*, the **derivative of *f*** is also a function denoted by *f'(x)* and defined as follows:

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The **domain** of this function is the set of all *x*-values for which the limit exists.

$$domain(f') \subseteq domain(f)$$

The Derivative as a Function

Interpretations of f':

1. The function $f'(x)$ tells us the **instantaneous rate of change** of $f(x)$ with respect to x at any value x in the domain of $f'(x)$.

2. The function $f'(x)$ tells us the **slope of the tangent** to the graph of $f(x)$ at any point $(x, f(x))$, provided x is in the domain of $f'(x)$.

The Derivative as a Function

Find the derivative of each function from **first principles** (i.e. using the limit definition).

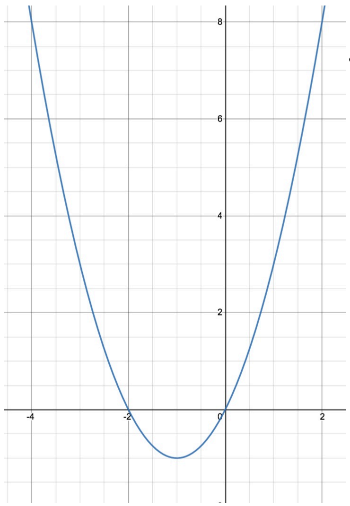
(a) $f(x) = x^2 + 2x$

(b) $h(x) = \sqrt{x + 3}$

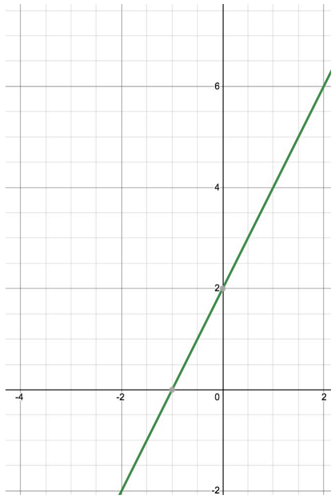
The Derivative as a Function

Example:

(a) $f(x) = x^2 + 2x$



f

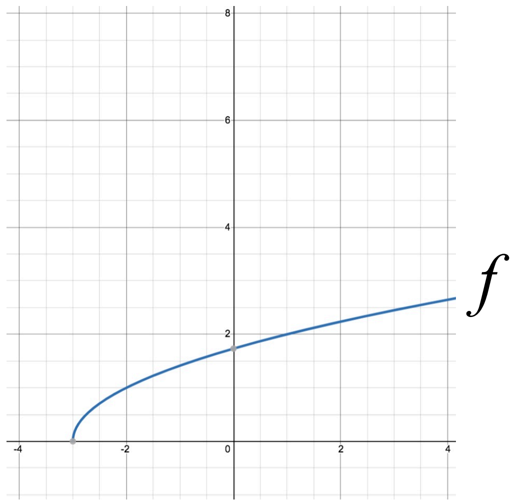


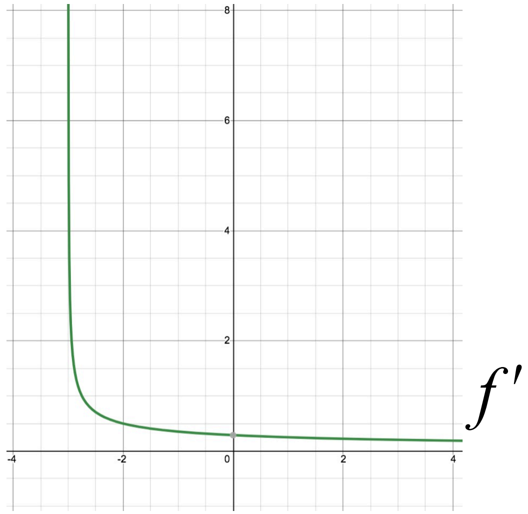
f'

The Derivative as a Function

Example:

(b) $h(x) = \sqrt{x+3}$



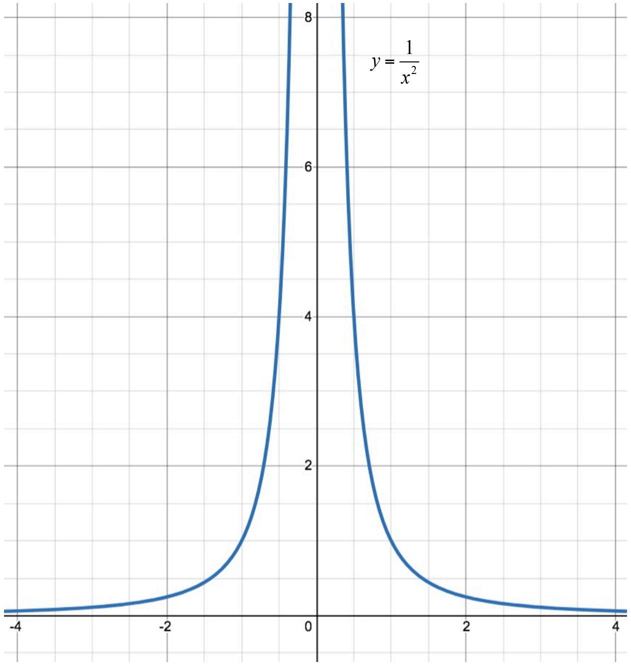


Using the Derivative

Example:

Consider the function $y = \dfrac{1}{x^2}$.

Find equations of the tangent lines at the points (-1,1) and (2, ¼).



Differentiable Functions

A function *f*(*x*) is said to be **differentiable** at *x*=*a* if we are able to calculate the derivative of the function at that point, i.e., *f*(*x*) is differentiable at *x*=*a* if

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

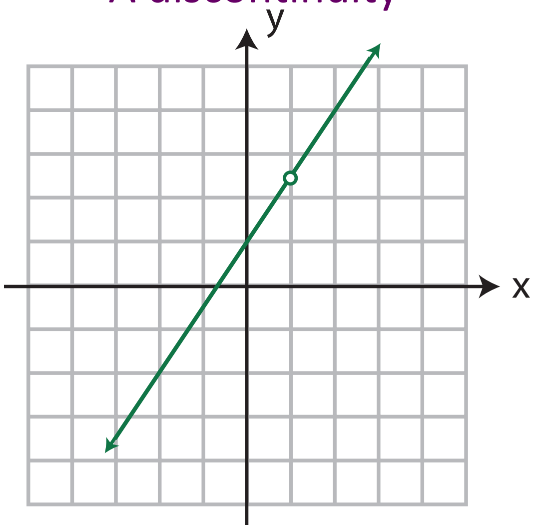
exists.

Differentiable Functions

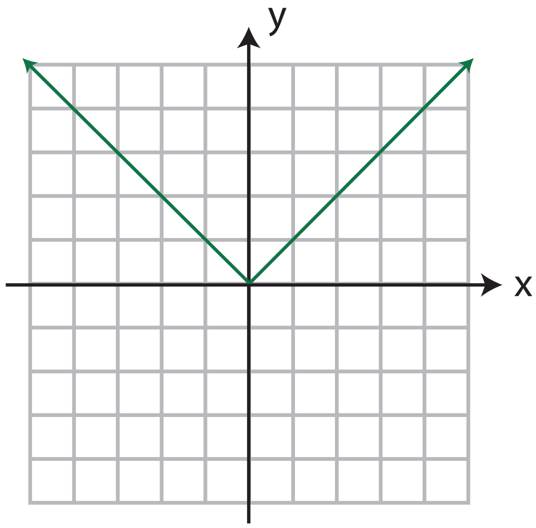
Geometrically, a function is differentiable at a point if its graph has a **unique tangent line with a well-defined slope** at that point.

3 Ways a Function Can **Fail** to be Differentiable:

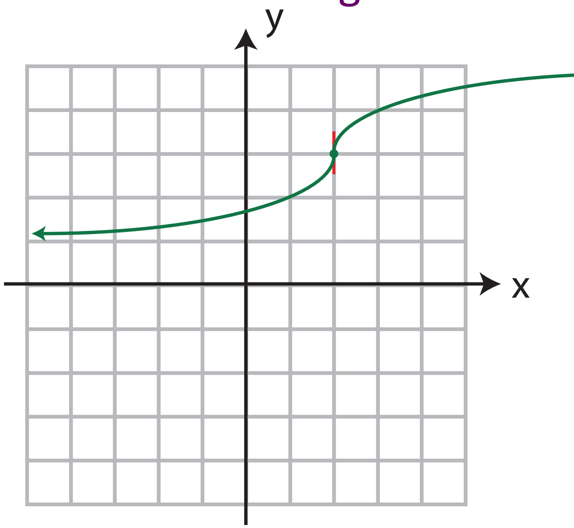
A discontinuity



A corner



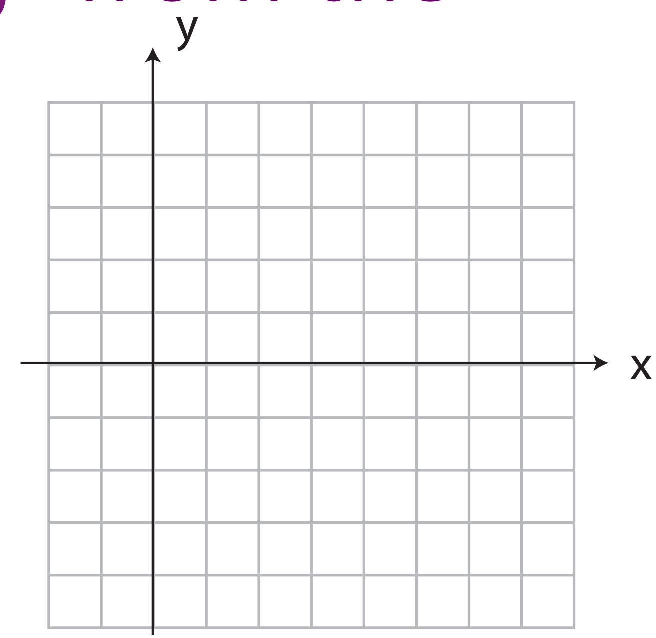
A vertical tangent



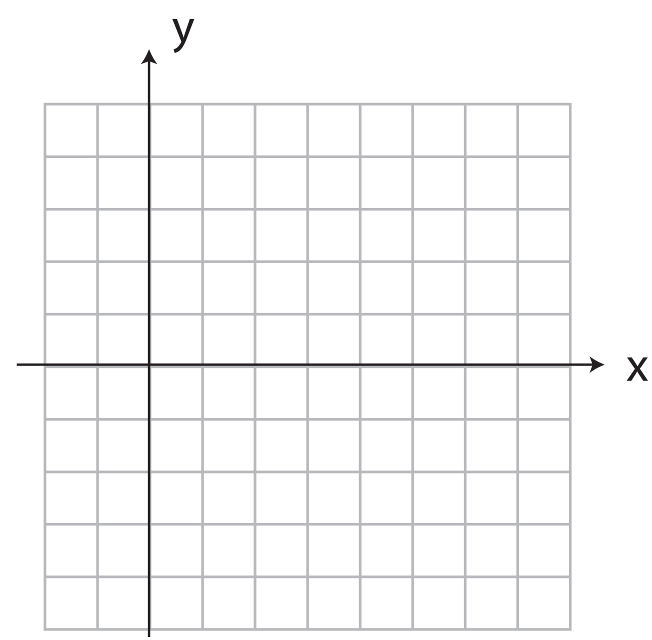
Sketching the Graph of *f'* from the Graph of *f*

Example:

(a) Sketch the graph of $f(x) = \left|x^2 - x\right|$.



(b) At which x-values is f(x) *not* differentiable.



(c) By looking at the graph of f(x), sketch the graph of f'(x).

Relationship Between Differentiability and Continuity

Theorem:
If f is differentiable at a,
then f is continuous at a.

This also means:
If f is not continuous at a,
then f is not differentiable at a.

If f is continuous at a, then f may or may not be differentiable at a.