

# Derivatives of Polynomial and Exponential Functions

## Section 3.1

# Basic Differentiation Rules

- All rules are proved using the definition of the derivative:

$$\frac{df}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- The derivative exists (i.e. a function is differentiable) at all values of  $x$  for which this limit exists.

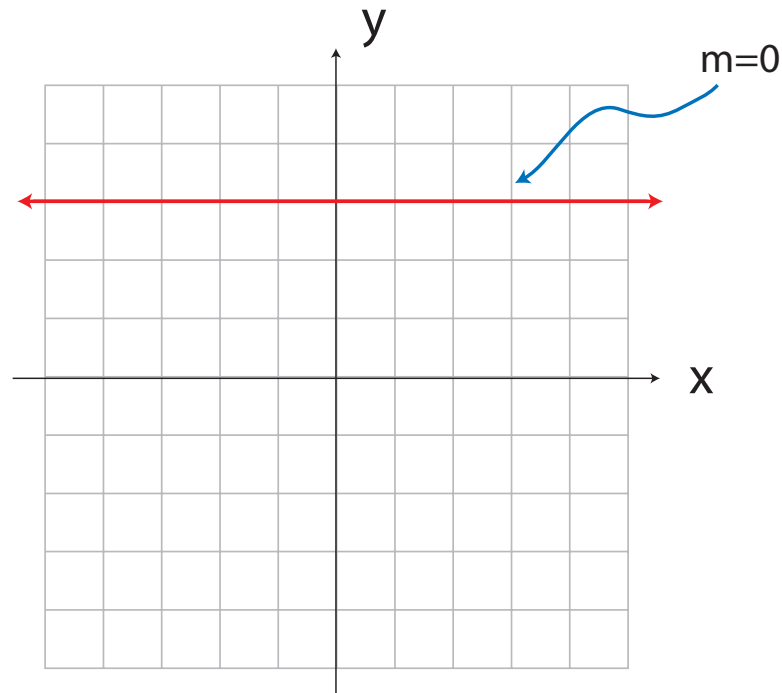
# The Constant Function Rule

If  $f(x) = k$ , where  $k$  is a constant, then  $f'(x) = 0$ .

**Example:**

$$f(x) = 3$$

$$f'(x) = 0$$



# The Power Rule

If  $f(x) = x^n$ , where  $n \in R$ , then  $f'(x) = nx^{n-1}$ .

## Example:

Differentiate the following.

(a)  $f(x) = x$

(b)  $g(x) = x^{100}$

(c)  $h(x) = \frac{1}{x^6}$

(d)  $s(t) = \sqrt{t}$

# The Constant Multiple Rule

Let  $c$  be a constant.

$$\text{Then } \frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)].$$

**Example:**

Find the derivative of each.

(a)  $f(x) = 3x^7$

(b)  $g(x) = \frac{\sqrt{5}}{x}$

# The Sum/Difference Rule

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

provided  $f$  and  $g$  are differentiable functions.

## Examples:

Differentiate.

(a)  $f(x) = 3x^4 + 5x^2 - 10$

(b)  $g(x) = \sqrt{5x} + \frac{x}{\sqrt{5}} - \pi^2$

# Derivative of the Natural Exponential Function

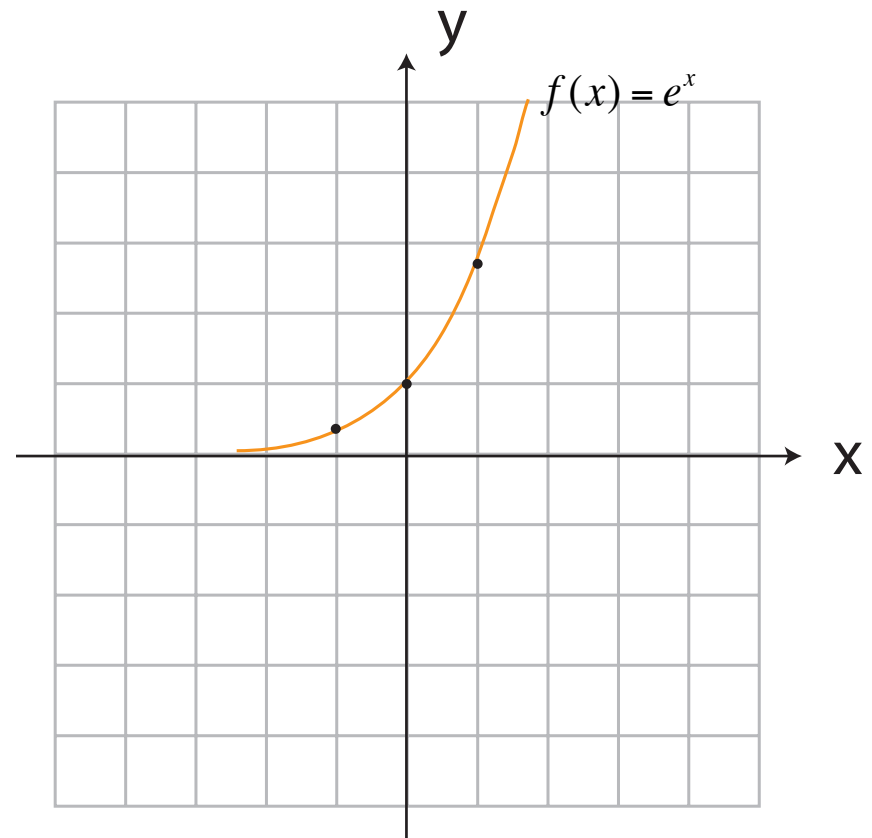
## Definition:

The number  $e$  is the number for which

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

Natural Exponential Function:

$$f(x) = e^x$$

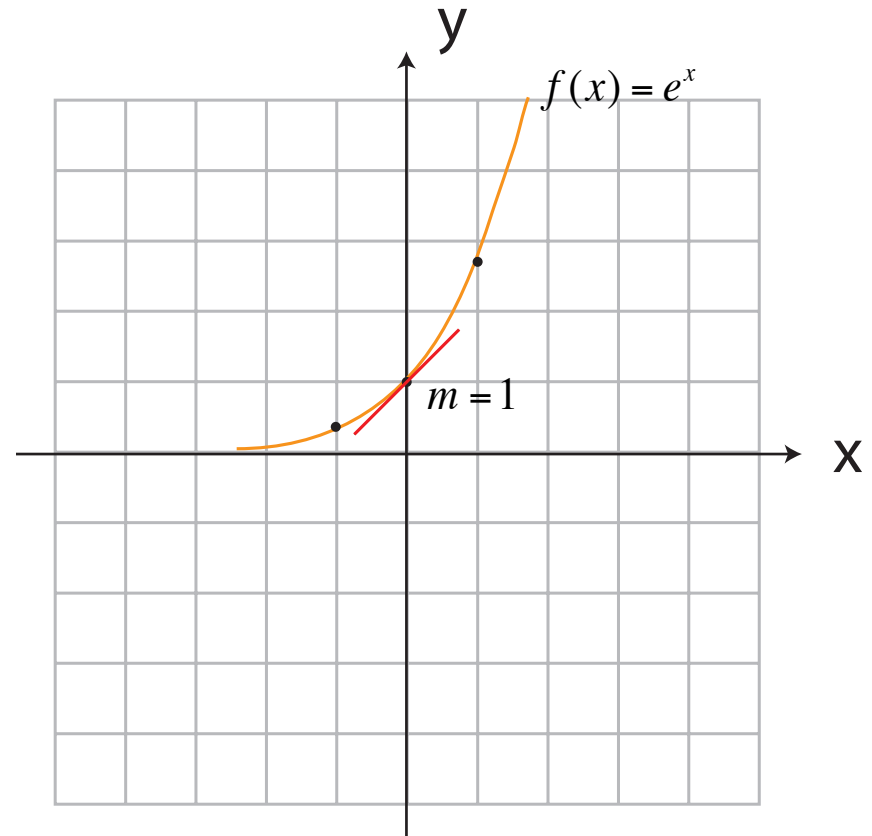


# Derivative of the Natural Exponential Function

## Note:

This definition states that the slope of the tangent to the curve at  $(0,1)$  is exactly 1, i.e.

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{e^{0+h} - e^0}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^h - 1}{h} \\ &= 1 \end{aligned}$$

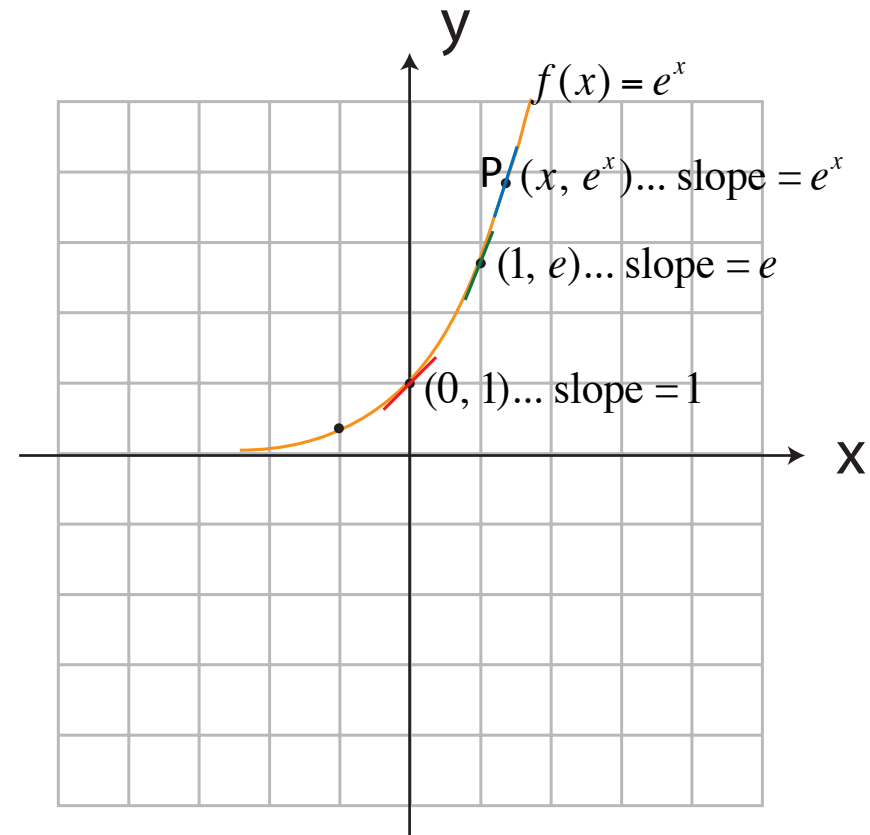


# Derivative of the Natural Exponential Function

If  $f(x) = e^x$ , then  $f'(x) = e^x$ .

***In words:***

The slope of the tangent line to the curve  $f(x) = e^x$  at the point P is equal to the value of the function at P.



# Exercise

**#60.** For what value of  $x$  does the graph of  $f(x) = e^x - 2x$  have a horizontal tangent?