Maximum and Minimum Values

Section 4.1

Maximum and Minimum Values

- Often the most important values of a function are its maximum and minimum values, i.e., its “extreme values” or “extrema”

Maximum Values

$f(c)$ is a <u>*global (absolute) maximum*</u> of f if $f(c) \geq f(x)$ for all x in the domain of f.

$f(c)$ is a <u>*local (relative) maximum*</u> of f if $f(c) \geq f(x)$ for all x in some interval around c.

Minimum Values

$f(c)$ is a *global (absolute) minimum* of f if $f(c) \le f(x)$ for all x in the domain of f.

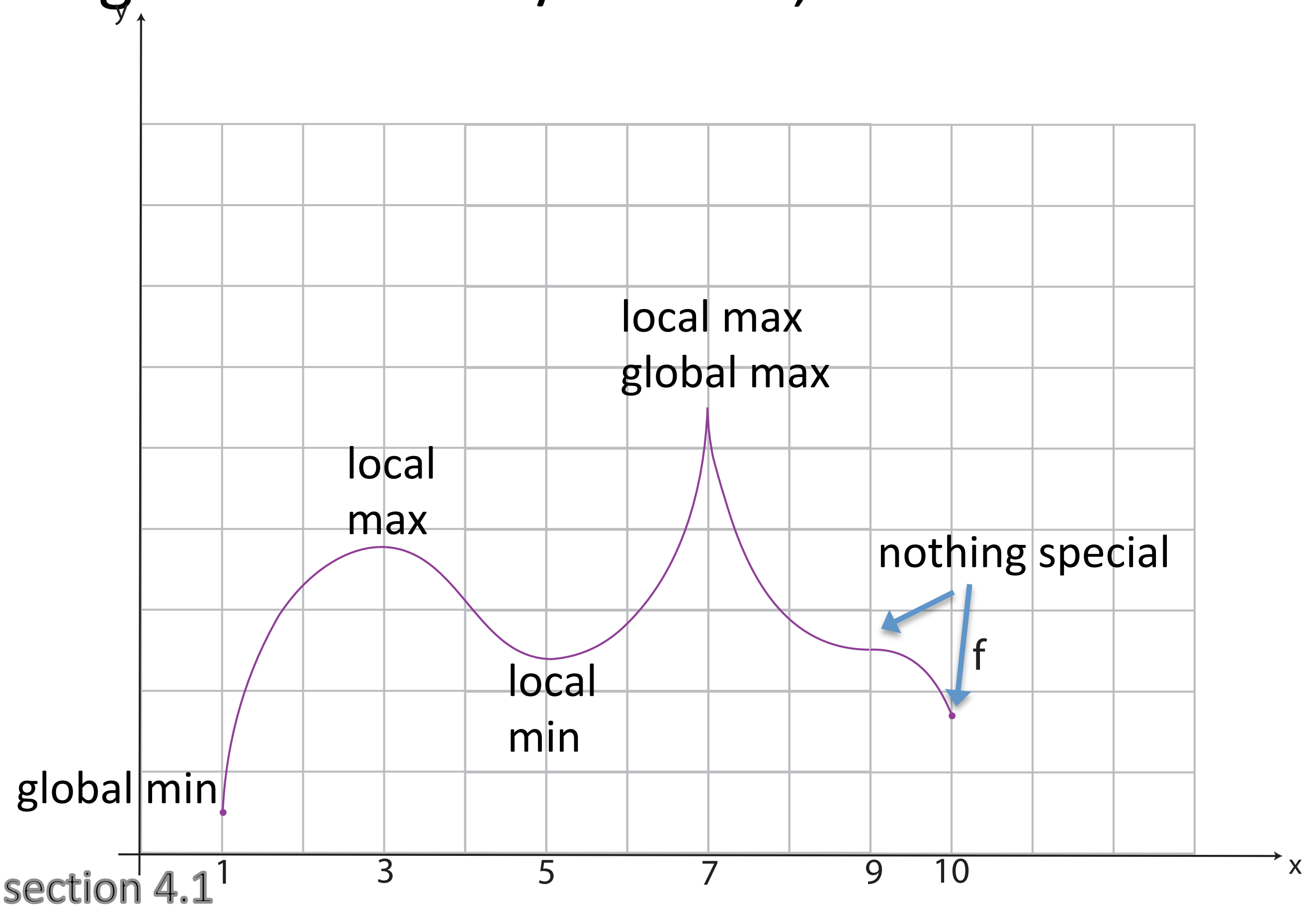
$f(c)$ is a *local (relative) minimum* of f if $f(c) \le f(x)$ for all x in some interval around c.

Exercise

Identify the labeled points as local maxima/minima, global maxima/minima, or none of these.

Exercise (Answers)

Identify the labeled points as local maxima/minima, global maxima/minima, or none of these.



Critical Numbers

Definition:
c is a **critical number** of *f* if *c* is in the domain of *f* and either ***f'*(c)=0** or ***f'*(c) does not exist**.

Fermat's Theorem:
If *f* has a local maximum or minimum at *c*, then *c* is a critical number of *f*.

Note:
This theorem says that if *c* is a critical number of *f*, then *f* ***might*** have an extreme value at *x*=*c*, but not necessarily.

Critical Numbers

Find the critical numbers of each function.

$(a) \quad f(x) = x^3 + 6x^2 - 15x$

$(b) \quad f(x) = x + \dfrac{1}{x}$

$(c) \quad f(x) = x^{1/3}(x-1)$

$(d) \quad f(x) = \dfrac{e^x}{x}$

Extreme Value Theorem

If $f(x)$ is **continuous** on a **closed interval** $[a,b]$, then *f* attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a,b]$.

The Closed Interval Method

To find the *absolute* maximum and minimum values of a continuous function *f* on a closed interval [a,b]:

1. Find the values of *f* at the critical numbers of *f* in (a,b).
2. Find the values of *f* at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

The Closed Interval Method

Find the absolute extrema of the following functions on the given intervals.

(a) $f(x) = x^3 + 3x^2 + 1, \quad [-1, 2]$

(b) $g(x) = |x - 3|, \quad [0, 4]$