What Derivatives Tell Us About the Shape of a Graph

Section 4.3

How Derivatives Affect the Shape of a Graph

- In studying a function, our goal is to find as much important information about it as possible (for example, the maximum and minimum value of the function).
- The derivative (and second derivative) can tell us information about $f(x)$ that we cannot see just by looking at the equation of $f(x)$.

Increasing/Decreasing Test

Recall:

A function *f* is called **increasing** on an interval I if

$$f(x_1) < f(x_2)$$

whenever $x_1 < x_2$ in I.

Increasing Test:

If *f'(x)* is positive for every *x* in some interval, then the function *f* is increasing on that interval.

Increasing/Decreasing Test

__Recall:__

A function *f* is called **decreasing** on an interval I if

$$f(x_1) > f(x_2)$$

whenever $x_1 < x_2$ in I.

Decreasing Test

If *f'(x)* is negative for every *x* in some interval, then the function *f* is decreasing on that interval.

Increasing/Decreasing Test

A function *f* can change from increasing to decreasing (or from decreasing to increasing) at either a critical number of *f* or at an x-value where *f* is discontinuous.

The First Derivative Test

(used to find local maxima and minima)

Suppose that *c* is a critical number of a continuous function *f*.

If f' changes from + to - at $x=c$, then *f* changes from increasing to decreasing at $x=c$ and $f(c)$ is a local maximum value.

If f' changes from - to + at $x=c$, then *f* changes from decreasing to increasing at $x=c$ and $f(c)$ is a local minimum value.

If f' does not change sign, then *f* doesn't have an extreme value at $x=c$.

Exercises

Find the intervals of increase and decrease and the local extrema for the following functions.

(a) $f(x) = 2x^3 + 9x^2 + 12x$

(b) $g(x) = \dfrac{1}{x^2}$

(c) $h(x) = x \ln x$

The Second Derivative

The derivative of the derivative is called the **second derivative**.

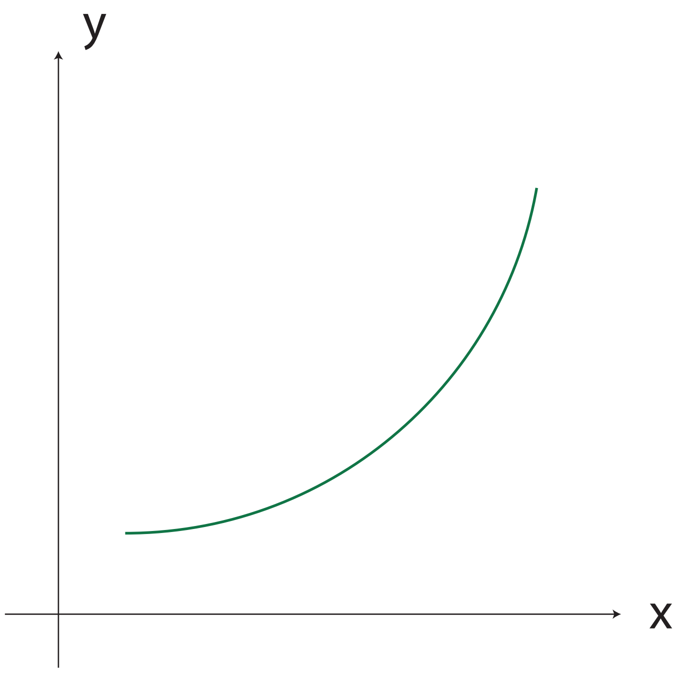
the second derivative of f = $f''(x) = \dfrac{d^2 f}{dx^2}$

Concavity

f'' provides information about f' and f:

When f'' is positive, f' is increasing, i.e., the rate at which f is changing is increasing.

When f'' is positive, the slopes of the tangents to the graph of f are increasing and the graph of f is **concave up.**

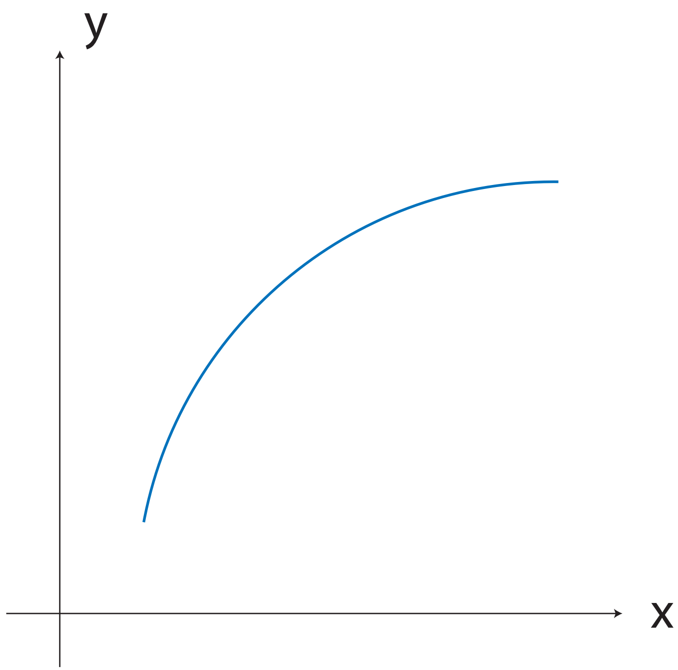


Concavity

f'' provides information about f' and f:

When f'' is negative, f' is decreasing, i.e., the rate at which f is changing is decreasing.

When f'' is negative, the slopes of the tangents to the graph of f are decreasing and the graph of f is **concave down.**

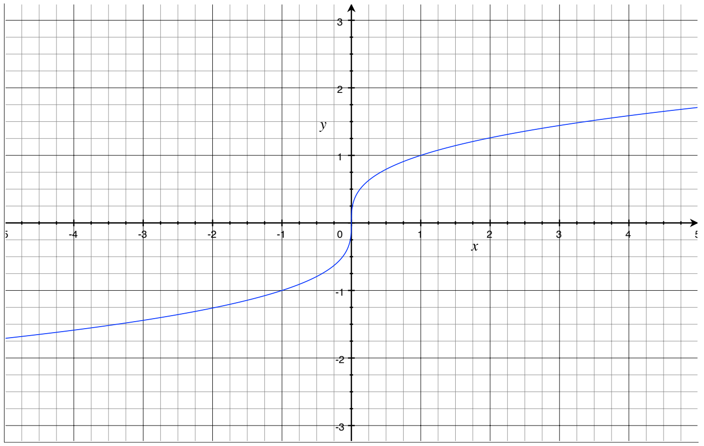


Inflection Points

When the graph of f changes concavity at a point *<u>in the domain of f</u>*, this point is called an **inflection point**.

<u>Note</u>:
At an inflection point, f''=0 or f'' D.N.E.

Example: $f(x)=x^{1/3}$



Exercises

Find the intervals of concavity and the inflection points for the following functions.

(a) $f(x) = x^3 + 6x^2 - 15x$

(b) $g(x) = xe^{-x}$

(c) $h(x) = \dfrac{x}{x+5}$

Second Derivative Test (also used to find Local Maxima and Minima)

Let x=c be a critical number of f.

If f''(c)>0 then the graph of f is concave up at x=c and f(c) is a local minimum value.

If f''(c)<0 then the graph of f is concave down at x=c and f(c) is a local maximum value.

If f''(c)=0 or f''(c) D.N.E. then the second derivative test doesn't apply and you have to use the other method.

Exercise

Find the local extrema of $f(x) = x \ln x$ using the second derivative test.