02/28/20 Test 2 Review

Test 2 2018

1a) $f(4) = f'(4) = f''(4)$
$= 0$

I) $f'(4) = 0$

No info indicates that

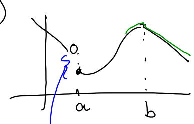
False

II) This true because $\rightarrow f'(4) = 0$ ✓

III) Inflection points
$f''(x) = 0$

False

1b)



I) This is a discontinuity $\Longrightarrow$ not diff.

II lim Left = lim = Right = value.
Continuous ✓

III) Not Diff ✗

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c) $x^2 y^4 = 1$

Take deriv on both sides

Product Rule : $\underline{x^2}_{f}\,\underline{y^4}_{g}$

$f'g + g'f$

$2x y^4 + 4y^3 \cdot y' x^2 = 0$

Point $(1,1) = (x,y)$

$2(1)(1)^4 + 4(1)^3 \cdot y'(1)^2 = 0$

$2 + 4y' = 0$

$4y' = -2 \rightarrow y' = -1/2$

Another!

Same as 1c point $(4,-1)$

$\sqrt{x} \cdot y^3 = 1$

Product Rule $\ast\ \sqrt{x} = x^{1/2}$

$\frac{1}{2}x^{-1/2} \cdot y^3 + 3y^2 y' \sqrt{x} = 0$

$\boxed{\ast\ \underline{\text{Note}}\quad \sqrt{x} \cdot y = 1 \qquad \frac{1}{2}x^{-1/2} y + y' \sqrt{x}}$

Plugging $(4,-1) = (x,y)$

$\frac{1}{2}(4)^{-1/2} \cdot (-1)^3 + 3(-1)^2 y' \sqrt{4} = 0$

$= \frac{-1}{2(4)^{1/2}} + 6y' = 0$

$= \frac{1}{2 \cdot 2} = \frac{1}{4}$

$-\frac{1}{4} + 6y' = 0$

$6y' = 1/4 \rightarrow y' = 1/24$

1d) $\frac{d}{dx} \ln(u) = \frac{u'}{u}$ *

$f(x) = Ax\ln(B+x)$
Product Rule

$f'(x) = A\ln(B+x) + Ax\frac{1}{B+x}$

$f'(0) = A\ln(B+0) + \cancel{A(0)\frac{1}{B+0}}$ 0

$f'(0) = A\ln(B)$

Another! Same question

$\frac{5x^{\cancel{2}}}{\ln(\cancel{x^3})x}$ NO!

$\frac{5x^2}{\ln(x^3)}$ ← f ← g Quotient Rule $\frac{f'g - g'f}{g^2}$

$\frac{u'}{u}$

$$\frac{10x\ln(x^3) - \frac{3x^2}{x^3}5x^2}{[\ln(x^3)]^2}$$

$$= \frac{10x\ln(x^3) - 15x}{[\ln(x^3)]^2}$$

* $\ln\left(\boxed{\frac{x^2+1}{x}}\right)$ u (Quotient Rule)

$\frac{1}{u} \cdot u'$

1e) $f(x) = \sin 2x$ at $x=0$

$f(0) = 0$

$f'(x)$	$2\cos(2x)$
$f''(x)$	$-4[\sin(2x)]$
$f'''(x)$	$-8[\cos(2x)]$

$f'(0)$	$2\cos(0) = 2\cdot 1 = 2$
$f''(0)$	$-4(\sin(0)) = 0$
$f'''(0)$	$-8[\cos(0)] = -8$

$T_3(x) = \cancel{f(x)}^{0} + \frac{f'(x)(x-a)'}{1!}$ $T_1(x)$

$+ \cancel{\frac{f''(x)(x-a)^2}{2!}}^{0} +$

$\frac{f'''(x)(x-a)^3}{3!}$

$T_1(x)$ $\frac{2x}{1!}$ I is wrong!

$T_3(x) = 2x - \frac{8x^3}{3!}$

2.) $g(x)$ concave down

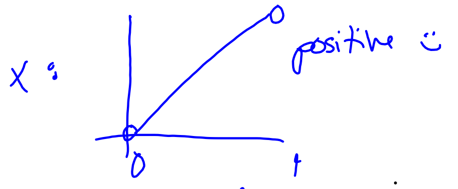
$\Longrightarrow$ $g''(x)$ is negative

same as $g''(x) < 0$

$g''(x) = x \ln(x)$ $\quad$ $(+)(-) = (-)$

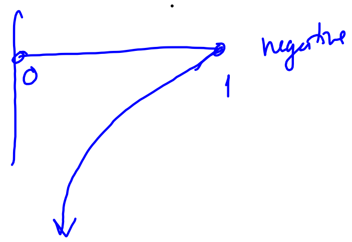
$\boxed{\text{True}}$

* look $(0, 1)$



$\ln(x)$

$\ln(1) = 0$



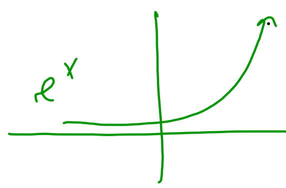
2b) Increasing functions $\Longleftrightarrow$ deriv is positive

Decreasing function $\Longleftrightarrow$ deriv is negative

$S(d) = e^{-d^2 - 0.1d - 0.2}$

* $\frac{d}{dx} e^u = e^u \cdot u'$

$S' = e^{-d^2 - 0.1d - 0.2} \cdot \underbrace{(-2d - 0.1)}_{-}$



$\boxed{\text{False}}$

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c) $m'(t) = \frac{\Delta m}{\Delta t} = \frac{kg}{d}$

$\boxed{\text{False}}$ $m'(t)$ not measured in kg.

3a (Modified)

Find $\lim_{h\to 0} \frac{1}{x+3}$

$* \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow \lim_{h\to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$

$\Rightarrow \lim_{h\to 0} \left[\frac{(x+3)}{(x+3)}\frac{1}{x+h+3} - \frac{1}{x+3}\frac{(x+h+3)}{(x+h+3)}\right]\frac{1}{h}$

Common $(x+h+3)(x+3)$

$= \lim_{h\to 0} \left[\frac{x+3-(x+h+3)}{(x+h+3)(x+3)}\right]\frac{1}{h}$

$\lim_{h\to 0} \left[\frac{\cancel{x}+\cancel{3}-\cancel{x}-h-\cancel{3}}{(x+h+3)(x+3)}\right]\frac{1}{h}$

$\lim_{h\to 0} \left[\frac{-\cancel{h}}{(x+h+3)(x+3)}\right]\frac{1}{\cancel{h}}$

$\lim_{h\to 0} \frac{-1}{(x+h+3)(x+3)} = \frac{-1}{(x+0+3)(x+3)}$

$= \frac{-1}{(x+3)(x+3)} = \boxed{\frac{-1}{(x+3)^2}}$

Using the lim Def find $f'(x)$ where $f(x) = \sqrt{x+2}$

$\lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$

$*$ multiply by $\frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}$

$\lim_{h\to 0} \frac{(x+h+2) - (x+2)}{h\left[\sqrt{x+h+2} + \sqrt{x+2}\right]}$

in general: $(a+b)(a-b) = a^2 - b^2$

$\lim_{h\to 0} \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h\left[\sqrt{x+h+2} + \sqrt{x+2}\right]}$

$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left[\sqrt{x+h+2} + \sqrt{x+2}\right]}$

$\lim_{h\to 0} \frac{1}{\left[\sqrt{x+h+2} + \sqrt{x+2}\right]}$

$= \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$

4) Taking the deriv of $R = \frac{K^{.96} L(\gamma+1)^2}{d^4}$

with respect to d

tells me that

K, L, γ are constants (wrt deriv)

$$\frac{K^{.96} L(\gamma+1)^2}{d^4} = R$$

looks like $\frac{5}{x} = 5x^{-1}$

Deriv $R' =$

$$K^{.96} L(\gamma+1)^2 \cdot \left(\frac{1}{d^4}\right)'$$

$$= K^{.96} L(\gamma+1)^2 \cdot (d^{-4})'$$

$$= K^{.96} L(\gamma+1)^2 (-4 \cdot d^{-5})$$

$$= \frac{-4K^{.96} L(\gamma+1)^2}{d^5} = R'$$

As one goes down the other goes up

(Knowledge b/c Deriv).

But the change gets smaller.

4b)

$$P(t) = 4.43\left(\frac{\pi}{2} + \arctan\left(\frac{t}{42}\right)\right)$$

$* \frac{d}{dx} \arctan(u) = \frac{1}{1+u^2} u'$

$$P(t) = \frac{4.43\pi}{2} + 4.43\arctan\left(\frac{t}{42}\right)$$

$$P'(t) = 0 + 4.43[\quad]$$

$$\frac{d}{dx} \arctan\left(\frac{t}{42}\right) = \frac{1}{1+\left(\frac{t}{42}\right)^2} \cdot \left(\frac{1}{42}\right)$$

$$T_1(x) = f(x) + \frac{f'(x)(x-a)}{1!}$$

$* \arctan(0) = 0$

$$T_1(x) = 4.43 \cdot \frac{\pi}{2} + \frac{4.43}{42}(x-a)$$

$$P'(t) = 4.43\left[\frac{1}{1+\left(\frac{t}{42}\right)^2} \cdot \frac{1}{42}\right]$$

$*$ Find T_2 (The quadratic approximation)

6) Finding critical points

$f(x) = (x^2-1)e^{-x^2}$

Prod Rule

$f'(x) = 2x\,e^{-x^2} + (x^2-1)e^{-x^2}\cdot 2x = 0$

$= e^{-x^2}\left[2x + -2x(x^2-1)\right] = 0$

NEVER $= 0$

$2x + -2x^3 + 2x = 0$

$= -2x^3 + 4x = 0$

$x(-2x^2+4) = 0$

$\boxed{x=0}$ or $-2x^2+4=0$

$= x^2-2=0$

$x^2 = 2 \rightarrow \boxed{x = \pm\sqrt{2}}$

EVT

Assumption:

f(x) is continuous on a closed interval [a,b].

Conclusion:

f(x) has an absolute max and an absolute min on [a,b].

Absolute Max/Mins of $f(x) = (x^2-1)e^{-x^2}$ on $[0,2]$

$x = -\sqrt{2}$ is not in [0,2] so don't check!

x	$f(x)$	
0	-1	min
2	$3e^{-4} \approx 0.055$	
$\sqrt{2}$	$e^{-2} \approx 0.135$	max

Find inflection points of $f(x) = e^{-x^2}$

$f'' = 0$

$f'(x) = e^{-x^2}\cdot 2x = \underbrace{2x}_{f}\,\underbrace{e^{-x^2}}_{g}$

$f''(x) =$ product Rule

$f' = -2$

$g' = e^{-x^2}\cdot -$

$-2e^{-x^2} + (e^{-x^2}\cdot -2x)\cdot -2x$

$= -2e^{-x^2} + 4x^2e^{-x^2} = 0$

f''

$= e^{-x^2}(-2+4x^2) = 0$

NEVER 0!

$-2+4x^2=0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

Concavity?

$f(x) = e^{-x}$

$f''(x)$ $f''(x)$ $f''($

$f''(-1) = +$ $f''(0) = -$ $f''(1)$

$-\frac{1}{}$ $\frac{1}{}$