MATHEMATICS 1LS3 TEST 2

Day Class

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Duration of Examination: 60 minutes

McMaster University, 29 October 2018

First name (PLEASE PRINT): ____________________

Family name (PLEASE PRINT): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE A PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	6	
4	6	
5	6	
6	6	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] It is known that $f(4) = 0$, $f'(4) = 0$ and $f''(4) = 0$. Which statement(s) is/are true for all functions $f(x)$ which satisfy these two conditions?

(I) $f(4) = 0$ is a local (relative) minimum of $f(x)$

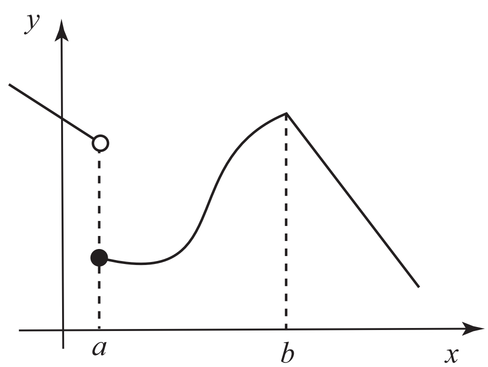
(II) the tangent line to the graph of $f(x)$ at $x = 4$ is $y = 0$

(III) $f(4) = 0$ is a point of inflection of the graph of $f(x)$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(b)[2] Identify all correct statements for the function $f(x)$ whose graph is given below.



(I) $f(x)$ is differentiable at a

(II) $f(x)$ is continuous at b

(III) $f(x)$ is differentiable at b

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

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(c)[2] The slope of the tangent to the curve given implicitly by $x^2y^4 = 1$ at the point $(1, 1)$ is

(A) 2	(B) -2	(C) 1	(D) -1
(E) $-1/4$	(F) $-1/2$	(G) $1/2$	(H) $1/4$

(d)[2] If $f(x) = Ax \ln(B + x)$, then $f'(0)$ is equal to

(A) A	(B) B	(C) AB	(D) $B \ln B$
(E) $AB \ln B$	(F) $AB \ln A$	(G) $A \ln B$	(H) $B \ln A$

(e)[2] Identify all correct Taylor polynomials of the function $f(x) = \sin 2x$ at $x = 0$.

(I) $T_1(x) = x$

(II) $T_3(x) = 2x - \dfrac{x^3}{3}$

(III) $T_3(x) = 2x - \dfrac{4x^3}{3}$

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

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2. **True/false questions: circle ONE answer. No justification is needed.**

(a)[2] Knowing that $g''(x) = x \ln x$, we conclude that the function $g(x)$ is concave down on $(0, 1)$.

TRUE FALSE

(b)[2] The linear-quadratic model for the percent S of cancer cells surviving radiation treatment states that

$$S(d) = e^{-d^2-0.1d-0.2}$$

where $d \geq 0$ is the dose (in Gray) per treatment of radiation.
$S(d)$ is an increasing function.

TRUE FALSE

(c)[2] Let $m(t)$ represent the mass of melting snow in kilograms, where t is the time in days. The units of $m'(t)$ are kilograms.

TRUE FALSE

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Questions 3-6: You must show correct work to receive full credit.

3. (a)[3] Find $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

(b)[3] Find $\lim_{x \to 0^+} x^4 \ln x$

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4. (a)[3] The resistance R of the flow of blood through a blood vessel (assumed to have the shape of a cylindrical tube) is given by

$$R = \frac{K^{0.96} L(\gamma + 1)^2}{d^4}$$

where L is the length of the tube, d is its diameter and $\gamma \geq 0$ is the curvature. The positive constant K represents the viscosity of the blood.

Find the derivative of R with respect to d and interpret your answer, i.e., explain what your answer implies for the dependence of R on the diameter of a blood vessel.

(b)[3] In the article *Phenomenological Theory of World Population Growth* by S. Kapitza, Physics-Uspekhi (39)1, we find the formula

$$P(t) = 4.43 \left(\frac{\pi}{2} + \arctan \frac{t}{42} \right)$$

where t is the time in years, with $t = 0$ representing 2007.

Find the linear approximation of $P(t)$ at $t = 0$. Round off all numbers to two decimal places.

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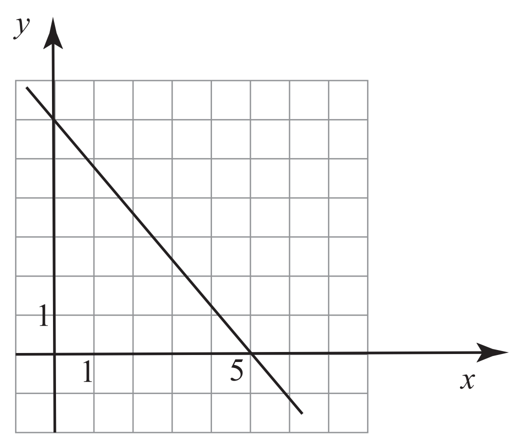
5. (a)[3] In *Hybrid equation/agent-based model of ischemia-induced hyperemia and pressure ulcer formation* by Alexey Solovyev et al., PLoS Computational Biology 9.5 (May 2013), the authors analyze the function

$$I(t) = I_{rest}\left(1 + ae^{-2t} + be^{-3t}\right)$$

where I_{rest} and a are positive constants, and the parameter b is negative. Find all critical numbers (t values only) of $I(t)$.

(b)[3] Let $h(x) = x\sin(f(x))$. The graph of $f(x)$ is a line shown below. Find $h'(5)$.



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6. (a)[2] Show that $f(x) = (x^2 - 1)e^{-x^2}$ has three critical points 0, $-\sqrt{2}$, and $\sqrt{2}$.

(b)[2] State the Extreme Value Theorem. Make sure to clearly identify assumption(s) and conclusion(s).

(c)[2] Find the absolute maximum and the absolute minimum of the function $f(x)$ from (a) on the interval $[0, 2]$.

THE END