MATHEMATICS 1LS3 TEST 3

Day Class
Duration of Examination: 60 minutes
McMaster University, 26 November 2018

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First name (PLEASE PRINT): ____________________

Family name (PLEASE PRINT): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE A PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	5	
4	5	
5	7	
6	7	
TOTAL	40	

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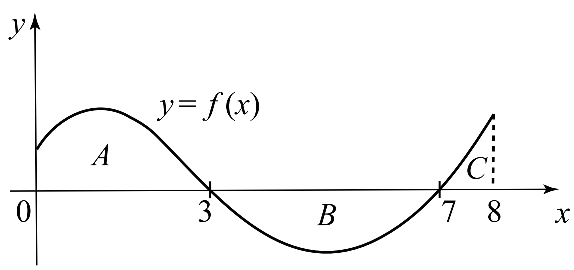
1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] $\displaystyle\int_1^2 \frac{1}{(x-3)^2}\,dx =$

(A) 0 (B) $-1/2$ (C) $-1/3$ (D) $1/2$

(E) $1/3$ (F) $3/2$ (G) $-3/2$ (H) $2/3$

(b)[2] In the graph below, the area of A is 4, the area of B is 6 and the area of C is 1 (in some units squared). Identify all correct statements.



(I) $\displaystyle\int_0^8 f(x)\,dx = -1$ (II) $\displaystyle\int_3^8 2f(x)\,dx = -5$ (III) $\displaystyle\int_3^7 (f(x)+3)\,dx = 6$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

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(c)[2] Which of the following definite integral(s) is/are positive? (Hint: Think! No need to calculate the integrals.)

(I) $\int_0^2 \cos x\,dx$ (II) $\int_0^3 \cos x\,dx$ (III) $\int_0^4 \cos x\,dx$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(d)[2] The average value of $f(x) = \sin x$ on $[0, \pi]$ is

(A) 0 (B) 1 (C) π (D) $\pi/2$

(E) $\pi/4$ (F) $2/\pi$ (G) $1/\pi$ (H) $\pi/8$

(e)[2] Which of the following improper integrals are *convergent*?

(I) $\int_1^\infty x^{-1.8}\,dx$ (II) $\int_1^\infty x^{-1}\,dx$ (III) $\int_1^\infty x^{-0.11}\,dx$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

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2. **True/false questions: circle ONE answer. No justification is needed.**

(a)[2] $P(t) = 9e^{0.1t}$ is a solution of the initial value problem $P'(t) = 0.9P(t)$, $P(0) = 10$.

TRUE FALSE

(b)[2] It is known that $\int_1^6 f(x)\,dx = -10$. Thus, $f(x) < 0$ for all x in $[1, 6]$.

TRUE FALSE

(c)[2] The left and the midpoint Riemann sums of $f(x) = x^{-1/3}$ on $[2, 12]$ satisfy $M_{15} < L_{15}$.

TRUE FALSE

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Questions 3-6: You must show correct work to receive full credit.

3. In December 2016, there was a notable increase in influenza cases (caused by the H3N2-like virus) in Winnipeg. Some researchers suggested that the number of influenza cases in Winnipeg could be modelled by

$$I'(t) = 240e^{-0.5t} - 40e^{-0.8t}$$

where t is time in days, with $t = 0$ representing 12 December 2016. On 12 December 2016, there were 230 reported cases of influenza in Winnipeg.

(a) [2] Estimate the number of influenza cases in Winnipeg on 14 December 2016 using Euler's Method with a step size of $\triangle t = 2$. Round your answer to the nearest integer.

(b) [3] Find a formula for $I(t)$ algebraically and use this formula to find the actual number of influenza cases in Winnipeg on 14 December 2016. Round your answer to the nearest integer.

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4. (a) [2] Find an approximation of the area of the region below the graph of $y = \ln x$ and over the interval $[1, 3]$, using a Riemann sum with 4 rectangles and right endpoints. Round your answer to three decimal places. Sketch the function and the four rectangles involved.

(b) [3] Find the exact area of the region in part (a) by evaluating $\int_1^3 \ln x \, dx$. Round your answer to three decimal places.

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5. (a)[3] Sketch (shade) the region bounded by the graphs of $y = \sqrt{x}$, $y = 1$, $x = 0$ and $x = 4$. Set up, but **do not evaluate**, the formula for its area. Your formula should not include absolute value.

(b)[4] Set up a formula for the volume of the solid obtained by rotating the region in part (a) about the y-axis. Your formula should not include absolute value. **Do not evaluate the integral.**

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6. A blood concentration of acetaminophen (common pain reliever) higher than 200 mcg/mL (micrograms per millilitre), reached 4 hours after ingestion, is known to increase the risk of liver damage. Even without taking a medication, a small amount of acetaminophen can be found in the body; consequently, we assume that the initial concentration is 8 mcg/mL. (Source: University of Rochester Medical Centre.)

Suppose that the concentration of acetaminophen in the blood of a patient, when following a certain protocol (such as after a minor surgery), changes according to $c'(t) = 40.2te^{-0.1t^2}$, measured in mcg/mL per hour. The dosing protocol (ingestion) starts when $t = 0$.

(a) [3] Find the indefinite integral $\int 40.2te^{-0.1t^2}\, dt$.

(b) [2] Determine whether a patient, subjected to the dosing protocol described above, faces an increased risk of liver damage.

(c) [2] What does the integral $\int_0^\infty 40.2te^{-0.1t^2}\, dt$ represent, and what are its units?

THE END