MATHEMATICS 1LS3 TEST 1

Day Class

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Duration of Examination: 60 minutes

McMaster University, 30 September 2019

First name (PLEASE PRINT): ____________________

Family name (PLEASE PRINT): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	4	
4	8	
5	8	
6	4	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] Start with the graph of $y = \cos x$. Scale (expand) the graph horizontally by a factor of 3 and then shift right the graph you obtained by 6 units. Finally, expand this graph vertically by a factor of 4. The graph you obtained is

(A) $y = \frac{1}{4}\cos\left(\frac{x+2}{6}\right)$ (B) $y = \frac{1}{4}\cos\left(\frac{x-2}{6}\right)$ (C) $y = 4\cos\left(\frac{x+6}{3}\right)$

(D) $y = 4\cos\left(\frac{x}{3} - 2\right)$ (E) $y = 4\cos\left(\frac{x}{3} + 6\right)$ (F) $y = 4\cos\left(\frac{x}{3} - \frac{2}{3}\right)$

(G) $y = \frac{1}{4}\cos\left(\frac{x+2}{3}\right)$ (H) $y = \frac{1}{4}\cos\left(\frac{x-6}{3}\right)$

(b)[2] The turbidity (denoted by T) is a measure of cloudiness or haziness in water, and is used to assess the quality of drinking water. It is calculated from the formula

$$T = k\,\frac{S \ln N}{d^2}$$

where N is the number of phytoplankton, S is the amount of sediment and d is the depth (k is a positive constant). If S and d double (and k and N do not change), then the turbidity T will change by a factor of

(A) 1/2 (B) 2 (C) 1/4 (D) 4

(E) 1/8 (F) 8 (G) 1/16 (H) 16

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(c)[2] The domain of the function $f(x) = \sqrt{e^{1-x} - 4}$ is:

(A) all real numbers (B) $(0, 1 - \ln 4)$ (C) $(-1, \ln 4)$ (D) $(-\infty, 1 - \ln 4]$

(E) $(0, 1 - \ln 4]$ (F) $(-1, \ln 4]$ (G) $(-\infty, 1 - \ln 4)$ (H) $(-\infty, 1)$

(d)[2] Identify all correct statements about the function $f(x) = \dfrac{x^2+1}{x^2-1}$.

(I) $f(x)$ is not defined when $x = 0$.
(II) $x = -1$ is a vertical asymptote of the graph of $f(x)$.
(III) $y = 1$ is a horizontal asymptote of the graph of $f(x)$.

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(e)[2] Find the range of the function $y = 1 + 4\arcsin(12x)$.

(A) $[-1, 1]$ (B) $[0, 2\pi]$ (C) $[-4, 4]$ (D) $[-3, 5]$

(E) $[-\pi - 1, \pi + 1]$ (F) $[-2\pi, 2\pi]$ (G) $[-2\pi + 1, 2\pi + 1]$ (H) $[-4\pi, 4\pi]$

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2. **True/false questions: circle ONE answer. No justification is needed.**

(a)[2] An oscillatory input (intensity function of a group of spiking neurons) is given by the formula $\hat{\lambda}_1(t) = v_0 + a\cos(2\pi f_m(t+d))$. The period of $\hat{\lambda}_1(t)$ is $2\pi f_m$.

TRUE FALSE

(b)[2] The line $y = \pi$ is a horizontal asymptote of the graph of $f(x) = 2\arctan(x^3 + 1)$.

TRUE FALSE

(c)[2] A substance started decaying exponentially in year 2000, and reached 50% of its original amount in 2010. By 2030, it will decay to 25% of its original amount.

TRUE FALSE

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Questions 3-6: You must show correct work to receive full credit.

3. Based on the density of a soil sample taken from a forest floor, scientists can determine the depth it came from, by using the formula

$$d = f(\rho) = -5 \ln \left(\frac{0.7}{\rho} - 0.8 \right)$$

In this formula, ρ is the density of a soil sample and d is the depth in metres (so $d = 0$ labels the surface, and $d = 3$ is 3 m below the surface).

(a)[1] In the above formula, d is a function of ρ. State (in one sentence) what question is answered by finding the inverse function of d.

(b)[3] Find a formula for the inverse function of d.

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4. Find each limit (or else say that the limit does not exist, and explain why).

(a)[2] $\lim_{x\to\pi^+} \frac{13x+1}{\sin x}$

(b)[3] $\lim_{x\to\infty} \Big(\ln(2x^3+4) - \ln(x^3 - x + 2)\Big)$

(c)[3] Consider the function

$$f(x) = \begin{cases} \dfrac{x-1}{x^3-x} & \text{if} \quad x < 1 \\ \dfrac{x}{4} & \text{if} \quad x \geq 1 \end{cases}$$

Then $\lim_{x\to 1} f(x) =$

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5. The survival rate (i.e., the chance of survival) $S(D)$ of clonogenic cells (cancer cells) exposed to a radiation treatment can be modelled by

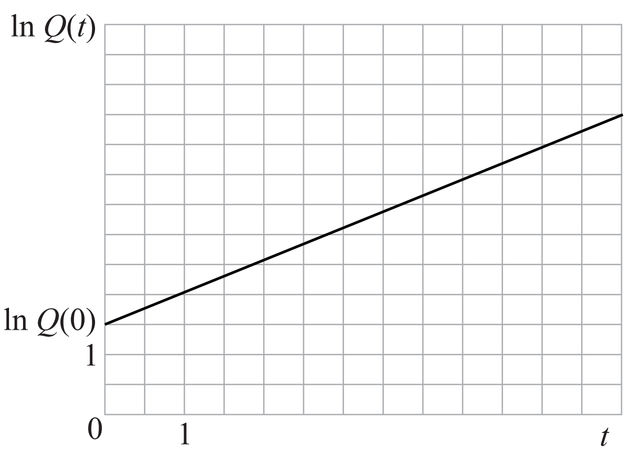
$$S(D) = e^{-0.5D^2 - D}$$

where $D \geq 0$ represents the applied radiation dose (measured in grays, Gy).

(a)[2] What is $S(0)$? Does it make sense?

(b)[3] Sketch the semilog graph (use ln) of the survival rate for $D \geq 0$. Label the axes.

(c)[3] The semilog graph below shows an exponentially increasing quantity $Q(t)$. Identify the point on the t axis which represents the time when the quantity quadruples (i.e., is four times larger than initially). Justify your answer.



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6. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

> the average of membrane potentials of neurons in the element, that is
>
> $$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$
>
> where N_e, N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

(a)[2] View V as a function of V_i. Describe its graph in words.

(b)[2] View V as a function of N_e. What is the limit of V as N_e increases beyond any bounds (i.e., as it approaches ∞)?

THE END