MATHEMATICS 1LS3 TEST 2

Day Class Duration of Examination: 60 minutes McMaster University, 28 October 2019 E. Clements, M. Lovrić, E. Miller

First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.:

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR EN-SURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	6	
4	7	
5	7	
6	4	
TOTAL	40	

1. Multiple choice questions: circle ONE answer. No justification is needed.

(a)[2] If $f(x) = \ln(ax) \ln(bx)$ then f'(1) is equal to

(A) $\ln a \ln b$	(B) $\ln(a+b)$	(C) $\ln(ab)$	(D) $\frac{\ln(a+b)}{a+b}$
(E) $\frac{\ln(ab)}{a+b}$	(F) $\frac{\ln a \ln b}{a+b}$	(G) $\frac{1}{ab}$	(H) $\frac{1}{a} + \frac{1}{b}$

(b) [2] The graph of the second derivative f''(x) of a function f(x) is given. Identify all true statements.



(I) x = 4 is a point of inflection of f(x)

(II) The graph of f(x) is concave up on (1, 4)

(III) x = 1 is a point of inflection of f(x)

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(c)[2] It is known that the function f(x) is defined for all real numbers, and its derivative is given by $f'(x) = \frac{(x-3) e^{-2x}}{(4-x)^{1/3}}$. Find all its critical points/critical numbers. (A) no critical points (B) 0 only (C) 3 only (D) 4 only (E) 0 and 3 (F) 0 and 4 (G) 3 and 4 (H) 0, 3 and 4

(d) [2] It is known that f(3) = 4 and f'(3) = 0 and f''(3) = -2. Identify all true statements for all functions f(x) which satisfy these three conditions.

- (I) f(3) = 4 is a local (relative) maximum of f(x)
- (II) the tangent line to the graph of f(x) at x = 3 is y = 0
- (III) the linear approximation of f(x) at x = 3 is $L_3(x) = 4$.

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(e)[2] If
$$f(x) = 2^{\ln x} + (\ln x)^2 + \ln 2$$
, then $f'(1)$ is equal to
(A) $1 + \ln 2$ (B) $2^{\ln 2}$ (C) $2^{\ln 2} + (\ln 2)^2$ (D) 0
(E) 1 (F) $\ln 2$ (G) $4 \ln 2$ (H) $2 \ln 2$

2. True/false questions: circle ONE answer. No justification is needed.

(a)[2] From $f''(x) = e^{-x}(3-x)$ we conclude that the graph of f(x) is concave down on the interval (0,3).

TRUE FALSE

(b) [2] The function f(x) has a horizontal tangent at x = 1. Therefore, it must have a local maximum or a local minimum at x = 1.

TRUE FALSE

(c)[2] If f(x) = g(x)h(x), then by the product rule, f''(x) = g''(x)h(x) + g(x)h''(x). TRUE FALSE

Questions 3-6: You must show correct work to receive full credit.

3. (a)[3] Let $h(x) = \sin(2f(x))$. The graph of f(x) is a line shown below. Find h'(6).



(b)[3] Find y'(0), if $\arcsin(xy) = x^3 + y^2 - 1$, and y(0) = 1.

4. (a)[3] In the article Migration behaviour of grizzly bears in Northern British Columbia: contribution to a modelling approach we find the formula

 $P(t) = \arctan(1.7t) + 4.7$

where t represents time. Next, we read "initially, $P(t) \approx 1.7t + 4.7$, which gives a linear relationship." Explain why this statement is correct. [Hint: Think in terms of the linear approximation at t = 0.]

(b)[4] A simple model of diffusion states that the concentration of a substance diffusing in air is given by

$$c(x) = e^{-x^2 + 1}$$

where x is the distance from the source. This formula is sometimes simplified using a quadratic approximation near x = 0. Find that approximation.

5. The function $c(t) = t^2 e^{-6t}$ has been used to model the absorption of a drug (such as morphine); c(t) is the concentration (in milligrams per millilitre, mg/mL) of the drug in the bloodstream, and $t \ge 0$ is time (in hours).

(a)[3] The function c(t) has two critical points/critical numbers such that $t \ge 0$. Find them.

(b)[2] Give a statement of the Extreme Value Theorem. Make sure to clearly identify assumption(s) and conclusion(s).

(c)[2] Find the absolute maximum and the absolute minimum values that the concentration c(t) reaches during the first hour after the drug is administered, i.e., over the interval [0, 1].

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6. Consider the function

$$f(x) = \begin{cases} \frac{x-1}{x^4 - x^2} & \text{if } x < 1\\ \frac{1}{4} & \text{if } x = 1\\ \frac{x}{2} & \text{if } x > 1 \end{cases}$$

(a)[2] Find $\lim_{x \to 1} f(x)$.

(b)[2] Is f(x) continuous at x = 1? Explain why or why not.