MATHEMATICS 1LS3 TEST 3

Day Class Duration of Examination: 60 minutes McMaster University, 25 November 2019 E. Clements, M. Lovrić, E. Miller

First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.:

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR EN-SURING THAT YOUR COPY OF THE PAPER IS COMPLETE. USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

Total number of points is 40. Marks are indicated next to the problem number. Calculator allowed: McMaster standard calculator Casio fx991MS or Casio fx991MS PLUS or lower Casio which has two lines of display and no graphing capabilities.

EXCEPT ON QUESTIONS 1 AND 2, you must show work to receive full credit.

Problem	Points	Mark
1	10	
2	6	
3	6	
4	8	
5	5	
6	5	
TOTAL	40	

1. Multiple choice questions: circle ONE answer. No justification is needed.

(a) [2] Which of the following improper integrals are convergent?

(I) $\int_{1}^{\infty} x^{-1.5} dx$	(II) $\int_1^\infty x^{-1} dx$	(III) $\int_{1}^{\infty} x^{-0.5} dx$	
(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(b) [2] Find
$$\lim_{x\to 0} \frac{e^{x^2} - 1 - x^2}{x^4}$$

(A) 0 (B) 1/5 (C) 1/3 (D) 1/2
(E) 1 (F) ∞ (G) 1/6 (H) 5/6

(c) [2] It is known that	t $\left(\frac{3x-1}{2x+1}\right)' = \frac{5}{(2x+1)^2}$	$\overline{)^2}$. What is the value of	of $\int_0^{1/3} \frac{5}{(2x+1)^2} dx$?
(A) 0	(B) 1/5	(C) 1/3	(D) 1/2
(E) 1	(F) 2	(G) 3	(H) 5

(d) [2] The value of
$$\int_0^1 \frac{4}{1+x^2} dx$$
 is
(A) 0 (B) 1 (C) $\pi/4$ (D) $\pi/2$
(E) 2π (F) π (G) $3\pi/4$ (H) $3\pi/2$

(e) [2] Which of the following numbers is/are positive?

	(I) $\int_{-1}^{1} e^{-3x} dx$	(II) $\int_{0.1}^{1} \ln x dx$ (4)	III) $\int_{-4}^{-2} x dx$
(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and II	I (H) all three

Name:	
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2. True/false questions: circle ONE answer. No justification is needed.

(a)[2] The temperature of a bottle of water placed in a fridge decreases proportionally to the difference between the temperature of the bottle and the fridge. A differential equation modelling this event would be described as a *pure-time* differential equation.

TRUE FALSE

(b) [2] The improper integral $\int_0^\infty e^{-2x} dx$ is convergent.

TRUE FALSE

(c)[2] The right and the left Riemann sums of $f(x) = x^{-1/3}$ on [2, 12] satisfy $R_{15} < L_{15}$. TRUE FALSE

Name:______ Student No.: ______

Questions 3-6: You must show correct work to receive full credit.

3. (a) [3] Find an antiderivative of $f(x) = \frac{\sqrt{\ln x}}{x}$.

(b)[3] In Wolf Science, 16 (7), 2018, we find a model for the change in the number of Eastern wolves in Algonquin Provincial Park

$$W'(t) = 2.2\sqrt{t} + 2e^{-0.1t}, \ W(0) = 140$$

Time t is measured in years, and t = 0 represents 1 July 2018.

Using Euler's method with the step size $\Delta t = 1$ estimate the number of wolves on 1 July 2020.

4. (a)[4] Sketch (shade) the region bounded by the graphs of $y = 3e^x$ and $y = e^{2x}$ on [0,4]. Write a formula for its area. Your answer should not contain absolute value. Do not evaluate the integral(s) involved.

(b)[4] Consider the region bounded by the graphs of $y = 10 - x^2$ and $y = x^2$. Write a formula for the volume of the solid obtained by revolving this region about the x-axis. Do not evaluate the integral(s) involved.

5. (a)[2] Find the Taylor polynomial $T_2(x)$ for $f(x) = e^x$ at x = 0.

(b)[1] Use your answer in (a) to find a polynomial approximation of e^{-x^2} .

(c)[2] Use the polynomial from (b) to find an approximation of $\int_0^1 e^{-x^2} dx$. Leave you answer as a fraction or round off to two decimal places.

6. The rate of change of the number of new individuals infected by a strain H2T1 of influenza virus in Hamilton in January 2018 has been modelled by the function $p(t) = 120te^{-0.1t}$. The variable t is time in days; the time t = 0 represents 15 January 2018.

(a)[3] Find
$$\int_0^4 120t e^{-0.1t} dt$$

(b)[2] What does your answer in (a) represent?