MATHEMATICS 1LS3 TEST 1

Evening Class — E. Clements
Duration of Test: 60 minutes
McMaster University — 23 May 2019

FIRST NAME (please print): ____________________

FAMILY NAME (please print): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number in square brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Multiple Choice and True/False.

Problem	Points	Mark
1	6	
2	6	
3	7	
4	4	
5	6	
6	5	
7	3	
8	3	
TOTAL	40	

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1. **True/false questions: circle ONE answer. No justification is needed.**

(a) [2] $\arcsin(-1) = \dfrac{3\pi}{2}$

TRUE FALSE

(b) [2] The average rate of change of $g(x) = \ln x$ from $x = 1$ to $x = 2$ is $\ln 2$.

TRUE FALSE

(c) [2] $\lim_{t\to\infty} 4.43\left(\dfrac{\pi}{2} - \arctan\dfrac{2007-t}{42}\right) = 4.43\pi$.

TRUE FALSE

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2. **Multiple Choice**. Clearly **circle** the one correct answer.

(a) [2] Turbidity T is a measure of cloudiness or haziness in water and is used to assess the quality of drinking water. It is known that turbidity is proportional to the natural logarithm of the number of phytoplankton N, proportional to the amount of sediment S, and inversely proportional to the square of the depth d. Which formula represents the turbidity? (k is a constant)

(A) $T = k\,\dfrac{Sd^2}{N}$ (B) $T = k\,\dfrac{S\ln N}{d^2}$ (C) $T = k\,\dfrac{\ln N}{Sd^2}$ (D) $T = k\,\dfrac{\ln N}{d^2}$

(E) $T = k\,\dfrac{Sd}{N}$ (F) $T = k\,\dfrac{Sd^2}{\ln N}$ (G) $T = k\,\dfrac{Sd}{\ln N}$ (H) $T = k\,\dfrac{S\ln N}{d}$

(b) [2] Consider the function $rect_l(x) = \begin{cases} 1/l & \text{if} \quad 0 \le x \le l \\ 0 & \text{if} \quad otherwise \end{cases}$, where $l > 0$.

Determine which of the following is/are true.

(I) $\lim\limits_{x\to l^-} rect_l(x) = 1/l$ (II) $\lim\limits_{x\to 0^+} rect_l(x) = 0$ (III) $\lim\limits_{x\to -l} rect_l(x) = 0$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(c) [2] Which of the following functions approach(es) 0 faster than x^{-1} as $x \to \infty$?

(I) $f(x) = e^{-0.1x}$ (II) $g(x) = 100x^{-0.1}$ (III) $h(x) = 0.2x^{-1.1}$

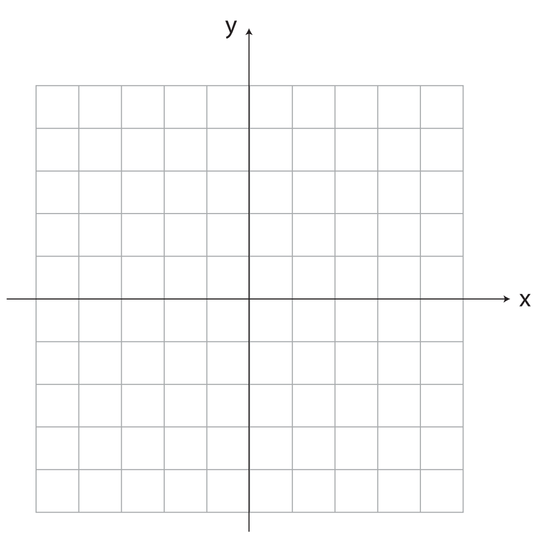
(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

3. (a) [2] Determine the domain of $f(x) = \sqrt{\dfrac{3x}{x+4}}$.

(b) [2] Find the range of $g(x) = -x^2 + 8x - 17$.

(c) [3] Sketch the graph of $f(x) = \begin{cases} \sqrt{1-x} & \text{if} \quad x < 1 \\ 2x - 3 & \text{if} \quad x \geq 1 \end{cases}$.



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4. Consider the Widmark formula for the Blood Alcohol Concentration estimation given by

$$C = \frac{A}{rW} - \beta t$$

(a) [2] Sketch a graph showing how the blood alcohol concentration C depends on the body weight W. Assume that all parameters are positive. Label the axes and any intercepts.

(b) [2] If person A's body weight is 25% greater than person B's, how does person A's blood alcohol concentration compare to that of person B's at time $t = 0$ (assuming all other parameters are equal)?

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5. A population changes according to the formula $P(t) = 1200e^{1.32t}$, where t is time in years.

(a) [1] In one sentence, state what question is answered by finding the inverse function.

(b) [2] Find the inverse function of $P(t)$.

(c) [3] Sketch the semilog graph (use ln) of the population $P(t)$ for $t \geq 0$. Label the axes.

6. A population of birds is modelled by the equation $B(t) = 2.5 \sin \frac{\pi}{6} t + 6$, where B is the number of birds in thousands at time t in months (January corresponds to $t = 0$).

(a) [2] State the maximum, minimum, and average number of birds in this population over the course of one year.

(b) [3] Sketch the graph of $B(t)$ for $0 \leq t \leq 12$.

7. [3] Use the definition of continuity to show that the function

$$f(x) = \begin{cases} \dfrac{x^2+3x-10}{x-2} & \text{if} \quad x \neq 2 \\ 8 & \text{if} \quad x = 2 \end{cases}$$

is not continuous at $x = 2$.

8. [3] Consider the function $f(x) = \sqrt{4-3x}$. Using the definition of the derivative, find $f'(x)$.

THE END