MATHEMATICS 1LS3 TEST 2

Evening Class Duration of Test: 60 minutes McMaster University E. Clements

11 June 2019

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.:

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR EN-SURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number in square brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit, except for Multiple Choice and True/False.

Problem	Points	Mark
1	6	
2	8	
3	7	
4	6	
5	6	
6	7	
TOTAL	40	

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1. True/false questions: circle ONE answer. No justification is needed.

(a) [2] $y(t) = 3e^{t^2-t}$ is a solution to the initial value problem y' = (2t-1)y, y(0) = 3.

TRUE FALSE

(b) [2] The definite integral $\int_{1}^{2} f(x) dx$ represents the total change in f(x) on [1, 2]. TRUE FALSE

(c) [2] Using integration by parts, we can write $\int \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx$. TRUE FALSE

2. Multiple choice questions: circle ONE answer. No justification is needed.

(a) [2] Let $g(x) = x^2 \sqrt{f(x)}$, where f is a differentiable function such that f(1) = 4 and f'(1) = 1. Find g'(1).

(A) 0	(B) - 4	(C) 1	(D) $-1/2$
(E) -1	(F) 17/4	(G) $15/2$	(H) none of these

(b) [2] Determine which of the statements is/are true for the function f(x) whose derivative f'(x) is drawn below.

- (I) f has three critical numbers
- (II) f has a local minimum at x = 3
- (III) f has an inflection point at x = 4



(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(c) [2] Consider the differential equation $f'(t) = \ln(t^2 + 1)$, where f(1) = 0. Using Euler's Method with step size 0.5, the approximate value of f(2) is

(A) 0.127	(B) 0.667	(C) 2.139	(D) 0.779
(E) - 0.597	(F) -0.882	(G) 0	(H) 0.936

(d) [2] Which of the following formulas is/are correct?

(I)
$$\int \ln |x| \, dx = \frac{1}{x} + C$$

(II)
$$\int \tan x \, dx = -\ln |\cos x| + C$$

(III)
$$\int \frac{1}{1+x^2} \, dx = \arctan x + C$$

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

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3. (a) [3] Find y'(0), if $\arcsin(xy) = x^3 + y^2 - 1$, and y(0) = 1.

(b) [4] Determine the interval on which the graph of $g(x) = 3xe^{-0.2x}$ is concave up.

4. (a) [3] Estimate the value of $e^{0.2}$ using a Taylor polynomial of degree 3 and a suitable base point. Round your answer to **two** decimal places.

(b) [3] Find the absolute maximum and minimum values of f(x) = |4x - 3| on [0, 1].

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5. (a) [3] Evaluate $\lim_{x \to \infty} x \sin(1/x)$.

(b) [3] Find an approximation of the area of the region below the graph of $y = \sqrt{1 - x^2}$ and over the interval [0, 1], using L_3 (i.e., left sum with three rectangles). Round your answer to **three** decimal places. Sketch the function and the three rectangles involved.

6. (a) [4] Assume that P(t) is the proportion (or percent) of women initially infected with human papillomavirus (HPV) who no longer have the virus at time t (t is measured in years). The rate of change of P(t) is modelled by the function

$$p(t) = 0.5 - 0.21t^{1.5} + 0.5e^{-1.2t},$$

where $0 \le t \le 2$. For what proportion/percent of young women will the virus be gone within one year? Round your answer to **two** decimal places.

(b) [3] Evaluate
$$\int_{0}^{1} \frac{x}{1+x^{2}} dx$$