# **Working With Functions**

- Review addition, subtraction, multiplication, division, and composition of functions on your own...
- Review transformations of graphs and inverse functions (we'll do a brief review here)

### **Inverse Functions**

The function  $f^{-1}$  is the inverse of f if  $f^{-1}(f(x)) = x$  and  $f(f^{-1}(x)) = x$ .

Each of  $f^{-1}$  and f **undoes** the action of the other.

Some examples:

## What Functions Have Inverses?

A function has an inverse if and only if it is a one-to-one function.

A function f is **one-to-one** if for every y-value in the range of f, there is exactly one x-value in the domain of f such that y=f(x).

### Horizontal Line Test

If every horizontal line intersects the graph of a function in <u>at most one</u> point, then the graph represents a one-to-one function.

# Finding the Inverse of a Function

<u>Algorithm:</u>

1.Write the equation y=f(x). 2.Solve for x in terms of y. 3.Replace x by  $f^{-1}(x)$  and y by x.

*Note*: The domain and range are interchanged

Example:

### **Temperature Conversion**

The relationship between degrees Celsius (C) and degrees Fahrenheit (F) is linear.

We know that  $0^{\circ}C$  corresponds to  $32^{\circ}F$  and  $100^{\circ}C$  corresponds to  $212^{\circ}F$ .

### **Temperature Conversion**

(a) Find the function that converts  ${}^{o}C$  to  ${}^{o}F$ .

<u>Note</u>: input is <sup>o</sup>C and output is <sup>o</sup>F

**Data Points:**  $(0^{\circ}C, 32^{\circ}F)$  and  $(100^{\circ}C, 212^{\circ}F)$ 

$$slope = \frac{\text{change in output}}{\text{change in input}} = \frac{\Delta F}{\Delta C} = \frac{212 - 32}{100 - 0} = 1.8$$

*Function*: F(C) = 1.8C + 32

### **Temperature Conversion**

(b) Find the function that converts  ${}^{o}F$  to  ${}^{o}C$ .

<u>Note</u>: input is <sup>o</sup>F and output is <sup>o</sup>C

#### **One approach:** Find the INVERSE of F(C):

**Note:** we do <u>not</u> interchange variables at the end since F and C have a physical meaning

(C): F - 32 = 1.8C $\frac{F - 32}{1.8} = C$  $\therefore C(F) = \frac{F - 32}{1.8}$ 

F = 1.8C + 32