Proportional and Inversely Proportional Relationships

Example: Body Mass Index (BMI)

$$BMI = \frac{m}{h^2}$$

Note:

BMI is ***proportional*** to mass. If a person's mass changes (and their height remains the same), then their BMI will change by the same amount.

Proportional and Inversely Proportional Relationships

If $m_{new} = 1.10 \times m_{old}$

Then $BMI_{new} = \frac{m_{new}}{h^2} = \frac{1.10 \times m_{old}}{h^2} = 1.10 \times BMI_{old}$

So a 10% increase in body mass results in a 10% increase in BMI.

Proportional and Inversely Proportional Relationships

Example: Body Mass Index (BMI)

$$BMI = \frac{m}{h^2}$$

Note:

BMI is *inversely proportional* to height squared. So an increase in height (with mass held constant), will result in a decrease in BMI.

Proportional and Inversely Proportional Relationships

If $h_{new} = 1.10 \times h_{old}$

Then

$$BMI_{new} = \frac{m}{h_{new}{}^2} = \frac{m}{(1.10 h_{old})^2} = \frac{1}{1.10^2} \times BMI_{old} \approx 0.83 \times BMI_{old}$$

So a 10% increase in height results in a 17% decrease in BMI.

Relationship Between Mass and Volume

The fundamental relation between the mass M and the volume V of an object, or living organism, states that

$$M = \rho V$$

where ρ is the density.

Note:
If density is constant, then mass is *proportional* to volume.

Linear Functions

- For a linear function, the **change in output** (Δy) is *proportional* to the **change in input** (Δx)

$$\Delta y \propto \Delta x \qquad \Rightarrow \qquad \Delta y = m \cdot \Delta x$$

- If the change in input is scaled by some factor, then the change in output is scaled by the same factor.
- We call this constant m the **slope** of the line

Linear Functions

slope:

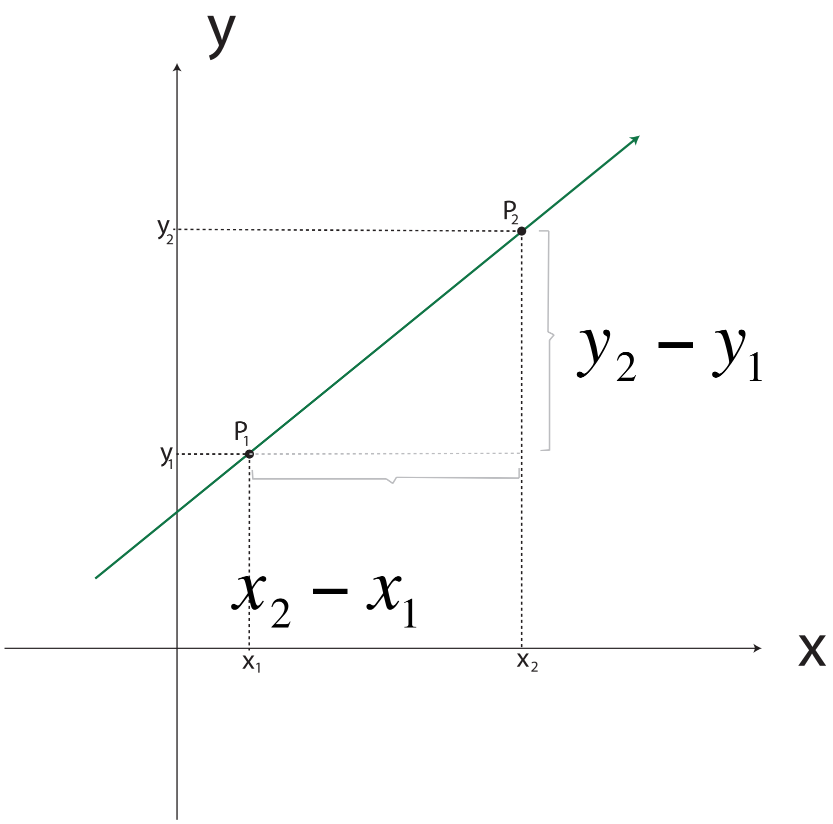
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

point-slope equation:

$$y - y_1 = m(x - x_1)$$

slope-y-intercept equation:

$$y = mx + b$$



Linear Model for the Population of Canada

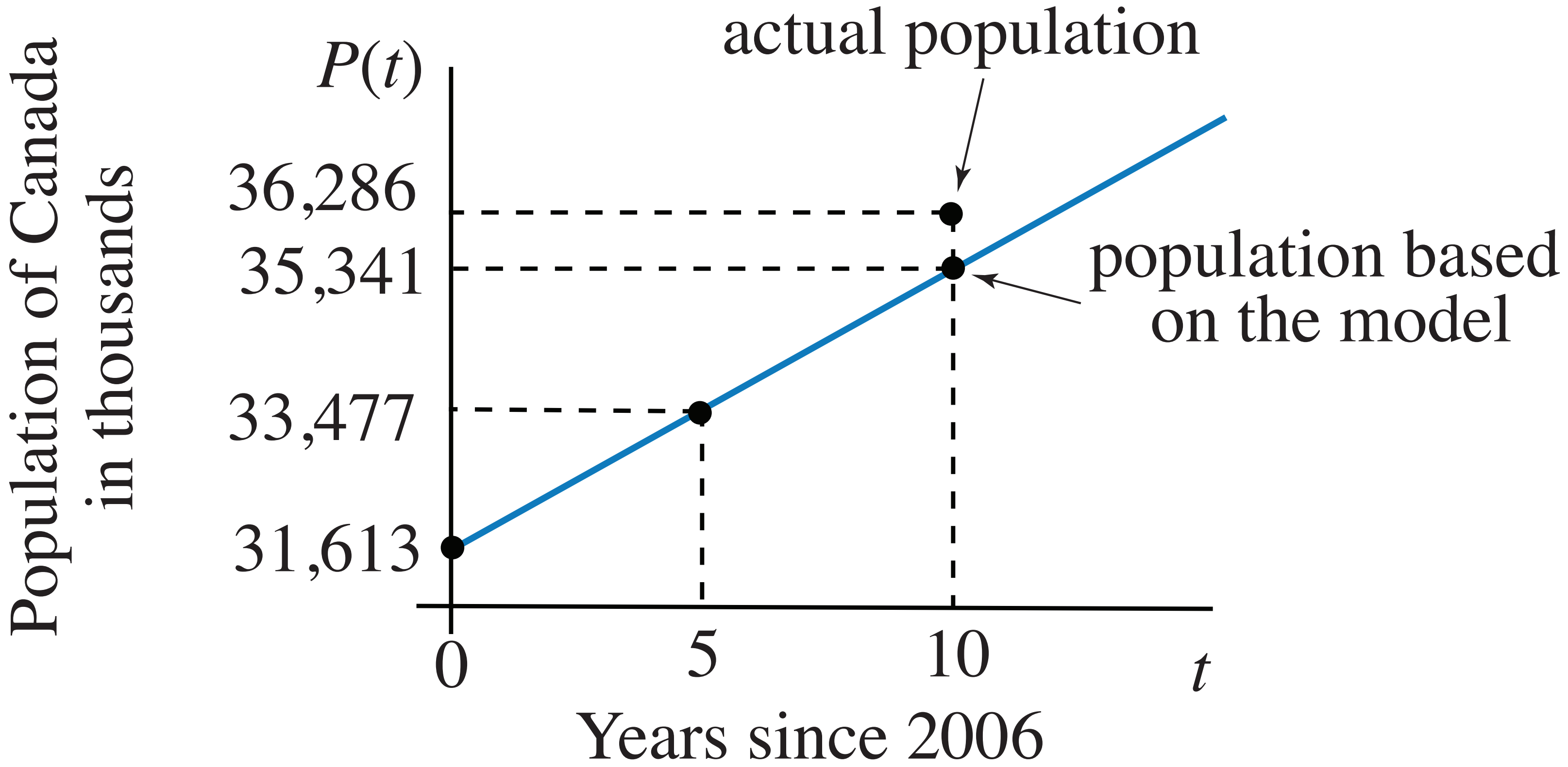
Data:

Year	Time, t	Population, P(t) (in thousands)
2006	0	31 613
2011	5	33 477
2016	10	36 286

(a) Create a linear model for the population of Canada as a function of time using the first two data points.

(b) Use this model to predict Canada's population in 2006.

Linear Model for the Population of Canada



Power Functions

A power function is a function of the form

$$f(x) = x^a$$

where a is a constant.

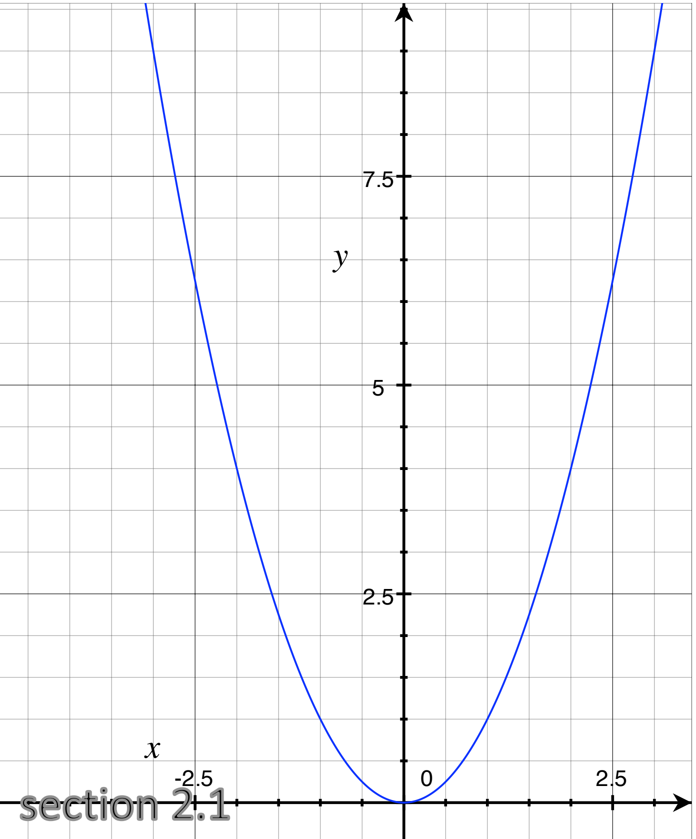
Note:

Although a can be any real number, we usually omit the case when $a = 0$.

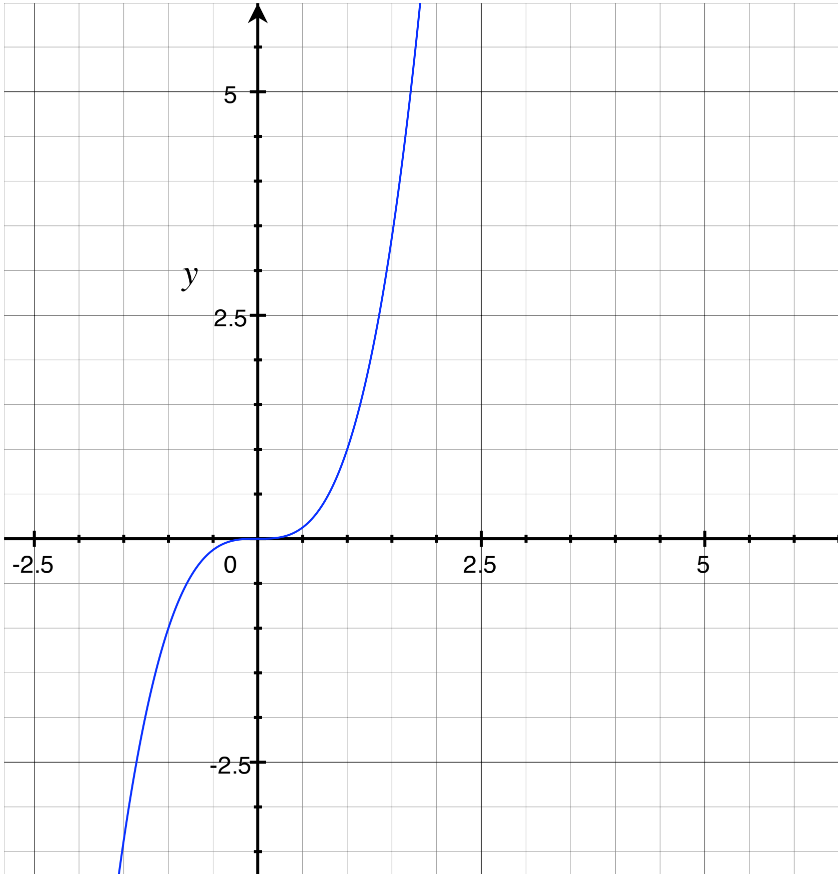
Power Functions

Some special cases:

a=2: $f(x) = x^2$



a=3: $f(x) = x^3$

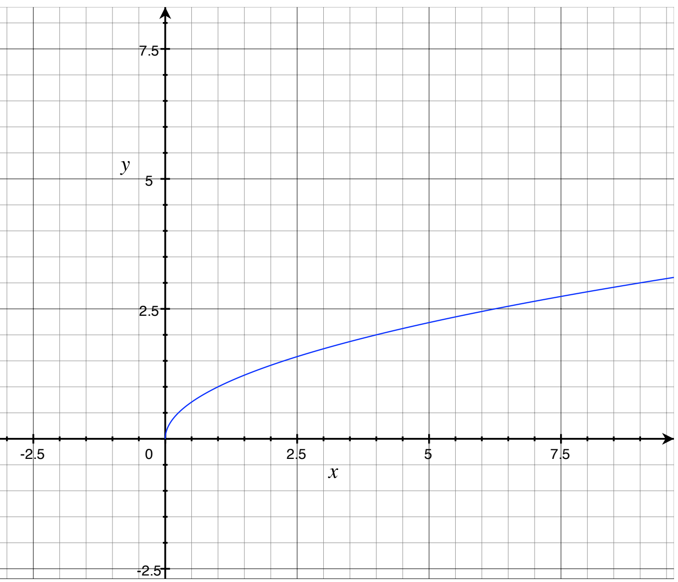


Power Functions

Some special cases:

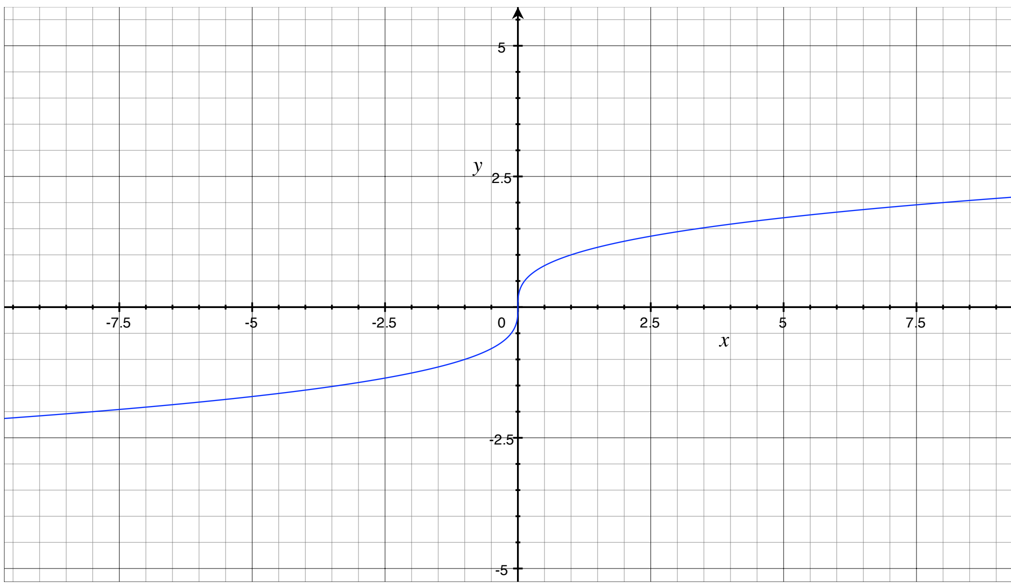
a=1/2: $f(x) = x^{\frac{1}{2}} = \sqrt{x}$

square root function



a=1/3: $f(x) = x^{\frac{1}{3}} = \sqrt[3]{x}$

cube root function



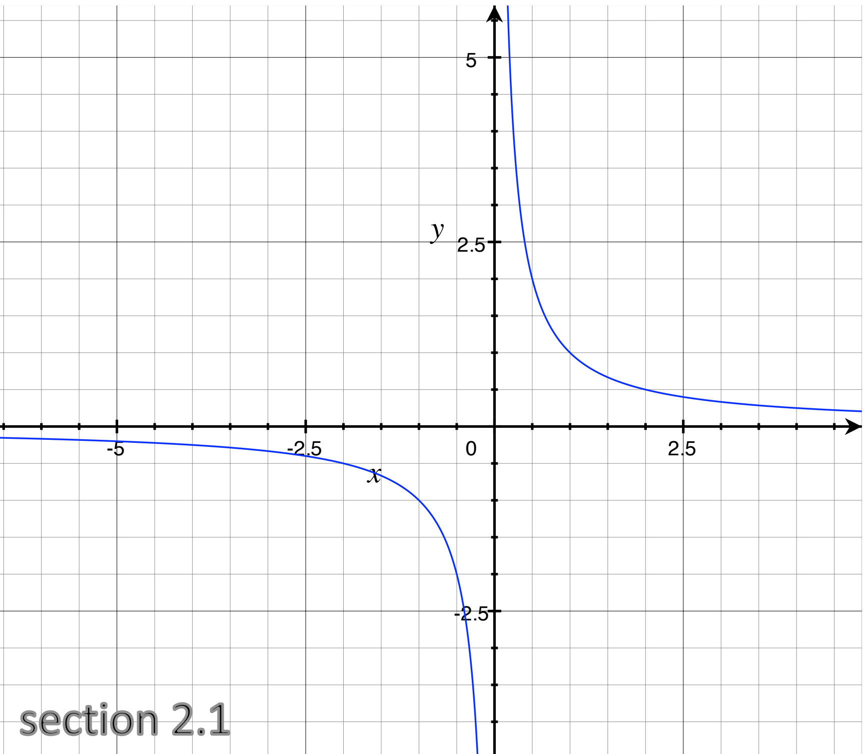
Power Functions

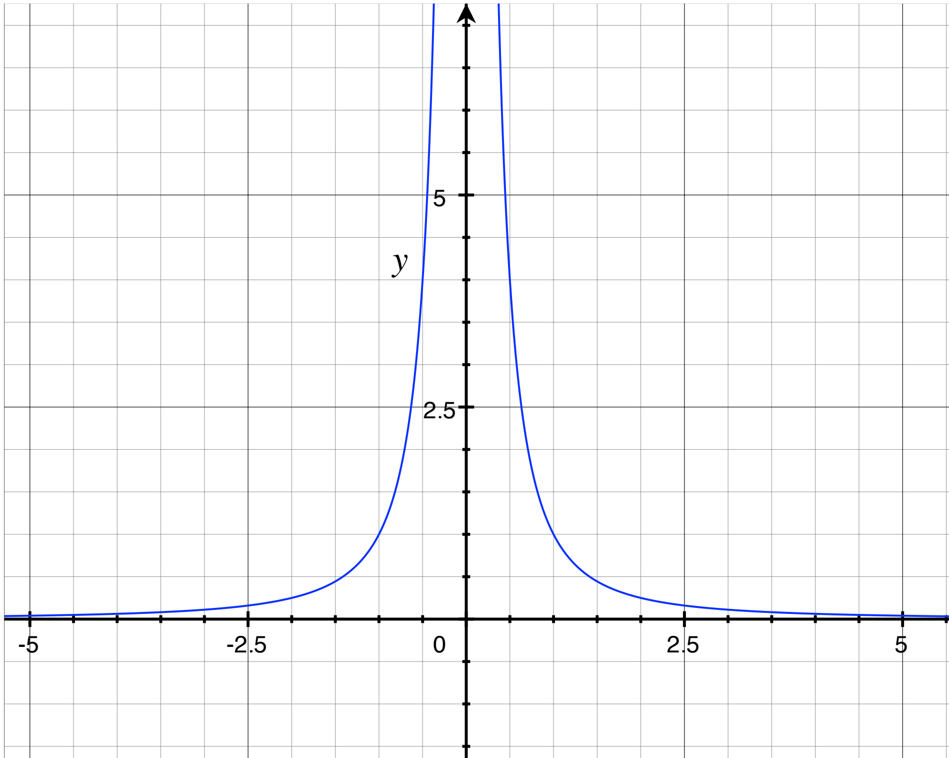
Some special cases:

a=-1: $f(x) = x^{-1} = \dfrac{1}{x}$

rational function

a=-2: $f(x) = x^{-2} = \dfrac{1}{x^2}$





Models Involving Power Functions

Example: Blood Circulation Time in Mammals

Blood circulation time is the average time needed for the blood to reach a site in the body and come back to the heart.

It has been determined that, for mammals, the blood circulation time is *proportional* to the fourth root of the body mass.

Models Involving Power Functions

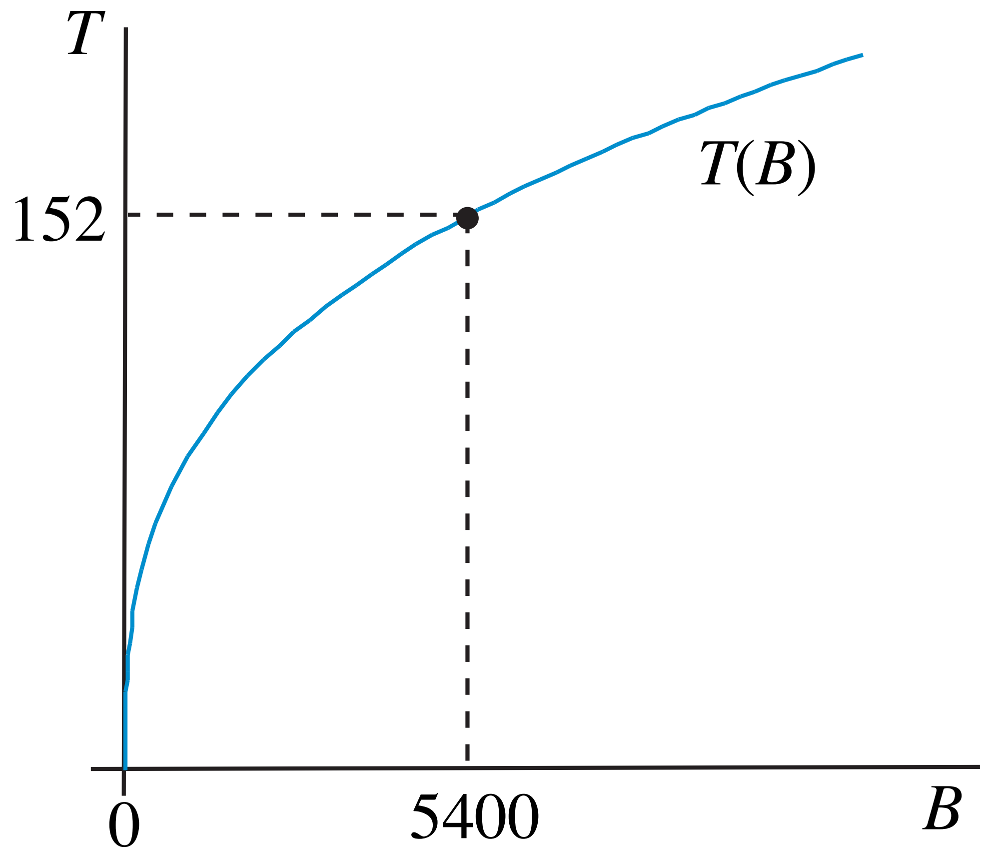
Example: Blood Circulation Time in Mammals

Model: $$T(B) = a\sqrt[4]{B}$$

where T is the blood circulation, in seconds, B is the body mass, in kilograms, and a is some proportionality constant.

Example: Blood Circulation Time in Mammals

Graph: $T(B) = a\sqrt[4]{B}, \qquad B \geq 0$



Models Involving Power Functions

If the body mass increases 10-fold, how does the blood circulation time change?

Models Involving Power Functions

This means that the blood circulation time of an elephant weighing 5400 kg is about 1.78 times longer than the blood circulation time of a cow that weights 540 kg.