Exponential Functions

An exponential function is a function of the form

$$f(x) = a^x$$

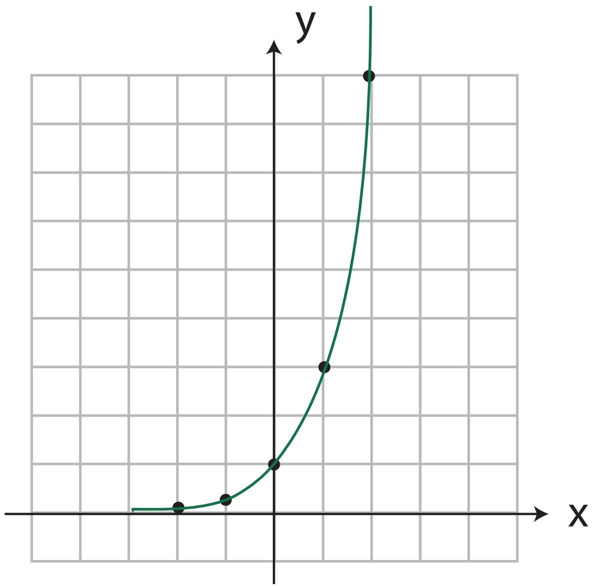
where a is a positive real number called the base and x is a variable called the exponent.

Domain: $x \in R$

Range: $y > 0$

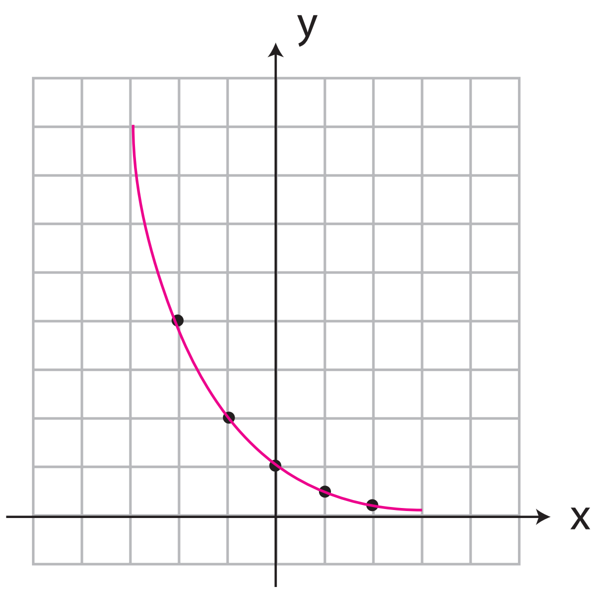
*Note: Please review EXPONENT LAWS on your own!

Graphs of Exponential Functions



$$f(x) = 3^x$$

When a>1, the function is increasing.



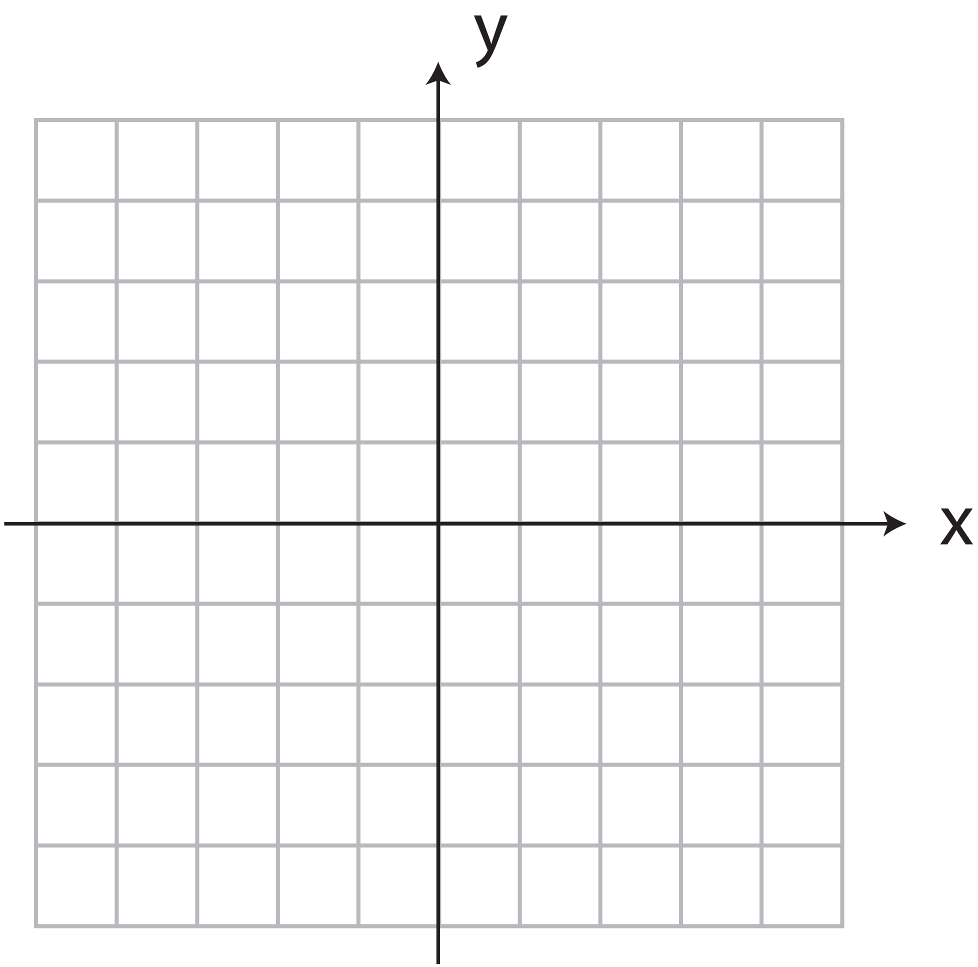
$$f(x) = \left(\tfrac{1}{2}\right)^x$$

When a<1, the function is decreasing.

y=0 is a horizontal asymptote

Transformation of an Exponential Function

Graph $f(x) = e^{-2x} + 3$.



__Recall__:

e is a special irrational number between 2 and 3 that is commonly used in calculus

Approximation:

$$e \approx 2.718$$

Logarithmic Functions

The inverse of an exponential function is a logarithmic function, i.e.

$$\boxed{\text{If } f(x) = a^x, \text{ then } f^{-1}(x) = \log_a x.}$$

Cancellation equations:

In general:

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

For exponentials & logarithms:

$$a^{\log_a x} = x$$

$$\log_a a^x = x$$

$$\boxed{\begin{array}{c} e^{\ln x} = x \\ \ln e^x = x \end{array}}$$

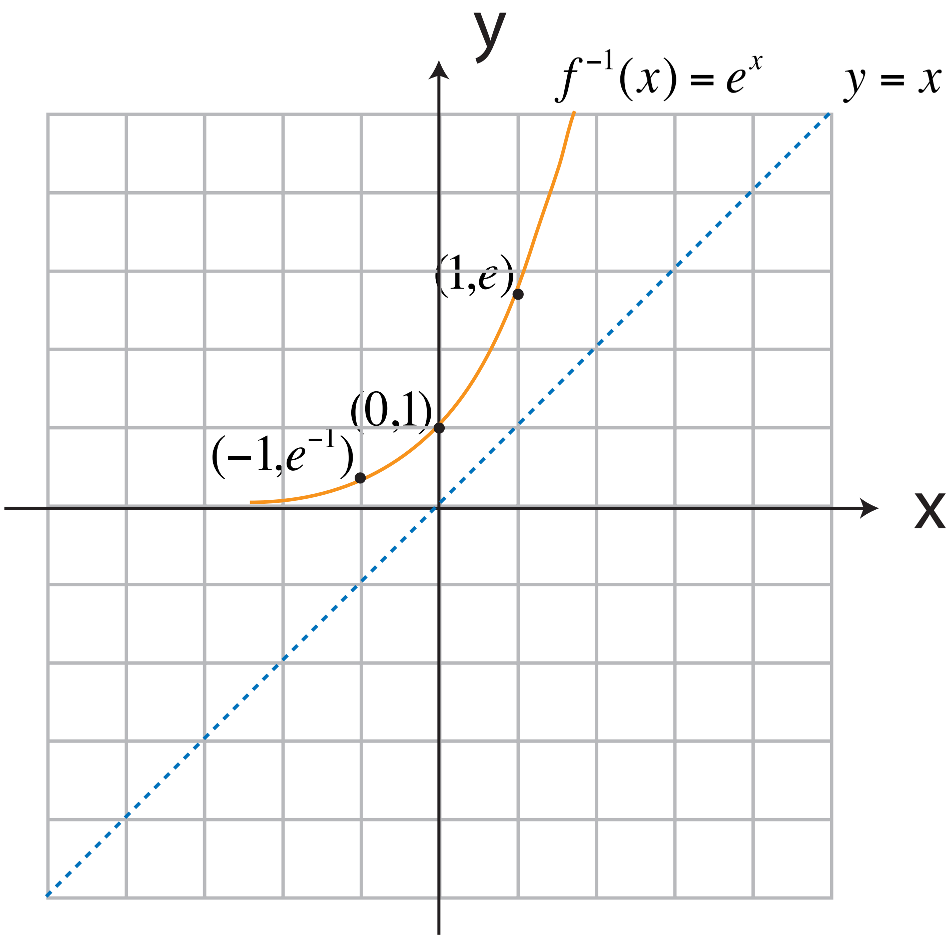
Graphs of Logarithmic Functions

Recall:
For inverse functions, the domain and range are interchanged and their graphs are reflections in the line $y = x$.

Example:
Graph $f(x) = \ln x$.

Graphs of Logarithmic Functions

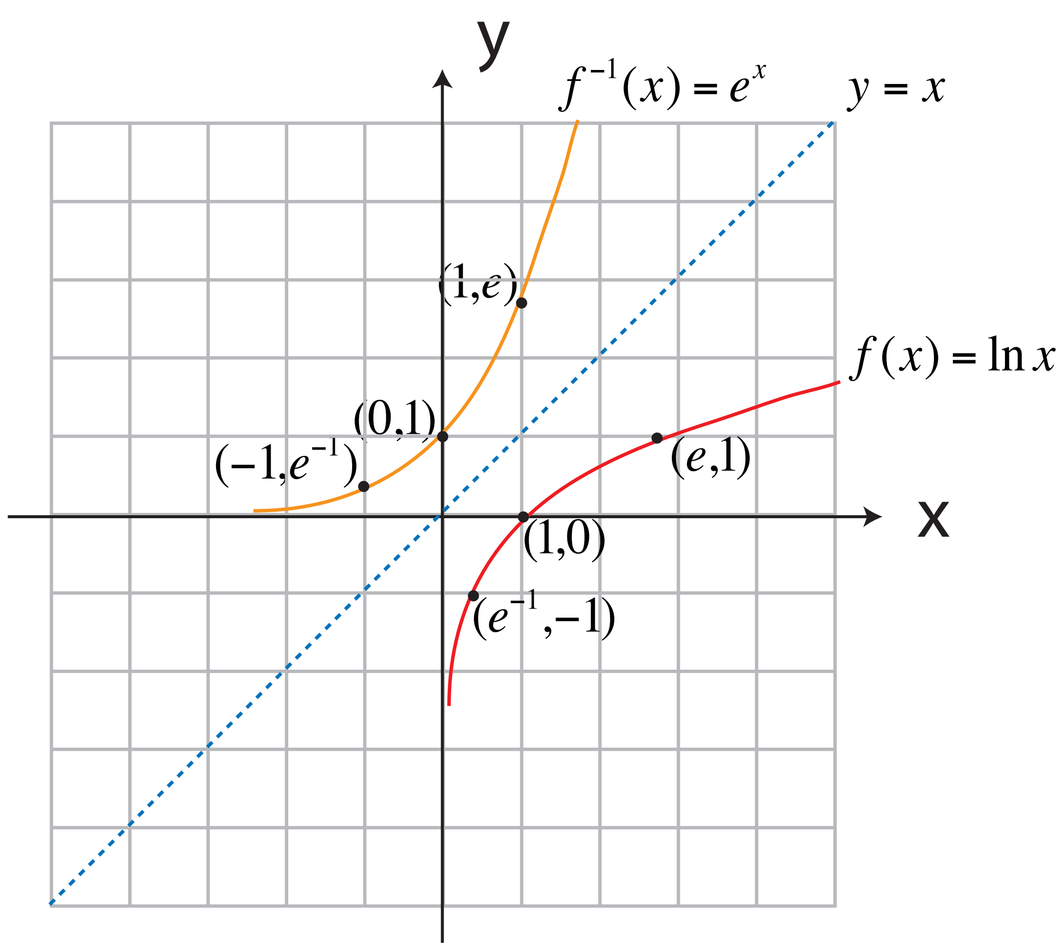
$e \approx 2.7$



Graphs of Logarithmic Functions

$e \approx 2.7$

Memorize!!!



Laws of Logs

For x,y>0 and p any real number:

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln(x^p) = p\ln x$$

Semilog Graphs

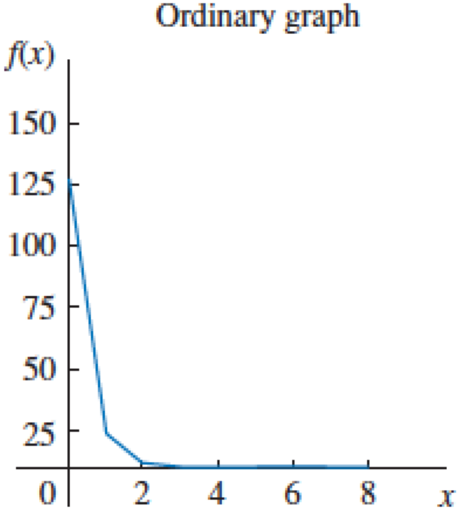
Definition:
A semilog graph plots the logarithm of the output against the input.

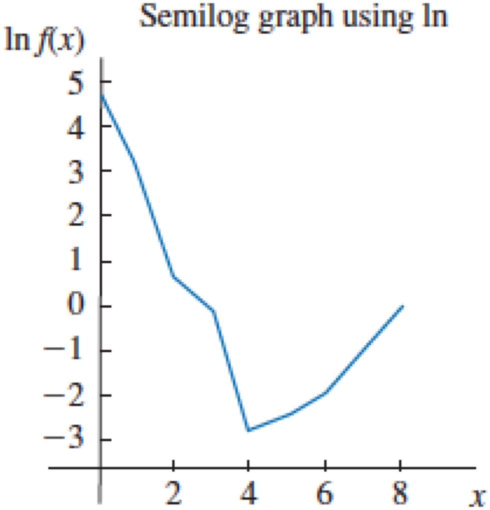
The semilog graph of a function has a *reduced range* making the key features of certain functions easier to distinguish.

Semilog Graphs

Example:

x	$f(x)$	$\ln f(x)$
0	120.12	4.79
1	24.34	3.19
2	2.19	0.78
3	0.89	−0.12
4	0.056	−2.88
5	0.078	−2.55
6	0.125	−2.08
7	0.346	−1.06
8	1.128	0.12





Semilog Graphs

Example:

Sketch the semilog graph of $f(x) = 10e^{-4x}$.

Double-Log Graphs

Definition:
A double-log graph plots the logarithm of the output against the logarithm of the input.

Semilog and Double-Log Graphs

Example: Blood Circulation Time in Mammals

Sketch the semilog and double-log graphs for the model $T(B) = 17.73B^{0.25}$.

Exponential Models

When the change in a measurement is proportional to its size, we can describe the measurement as a function of time by the formula

$$S(t) = S(0)e^{\alpha t}$$

where

$S(t)$ is the value of the measurement at time t

$S(0)$ is the initial value of the measurement, and

α is a parameter which describes the rate at which the measurement changes

Doubling Time

Example:

A bacterial culture starts with 100 bacteria and after 3 hours the population is 450 bacteria.

Assuming that the rate of growth of the population is proportional to its size, find the time it takes for the population to double.

Half-Lives of Drugs

	Half-life
Tetrahydrocannabinol … Marijuana (infrequent users)	1.3-3 days
Marijuana (frequent users)	1-10 days
Marijuana (if taken orally as pills)	25-36 hours
Marijuana (smoking/inhaling)	1.6-59 hours
LSD (Lysergic acid diethylamide)	3-5 hours
MDMA … ecstasy Methylenedioxymethamphetamine	6-10 hours
Caffeine adults	4-5 hours
Caffeine infants	10-20 hours
Caffeine with oral contraceptives	5-10 hours
Caffeine (if pregnant)	9-11 hours
Caffeine (liver disease)	several days
Codeine (Tylenol 3)	3-6 hours
Demerol (pain killer)	3-5 hours
Morphine (pain killer)	2-3 hours
Heroin (IV or inhaled)	3-5 minutes
Cocaine (benzoylmethylecgonine)	1 hour
Psilocin … magic mushrooms, shrooms	2-3 hours
Phencyclidine … rocket fuel, killer weed, angel dust	7-46 hours

Half-Lives of Drugs

Example: Thinking in Half-Lives

# of half-lives	amount left in body	% amount left in body
0	$M(0)$	100
1	$0.5M(0)$	50
2	$0.5^2M(0)$	25
3	$0.5^3M(0)$	12.5
4	$0.5^4M(0)$	6.25
5	$0.5^5M(0)$	3.125

** Many drugs are not effective when less than 5% of their original level remains in the body.