Trigonometric Functions

Trigonometric functions are used to model quantities that **oscillate**.

Trigonometric Models

Example: Seasonal Growth
A population of river sharks in New Zealand changes periodically with a period of 12 months. In January, the population reaches a maximum of 14, 000, and in July, it reaches a minimum of 6, 000.

Using a trigonometric function, find a formula which describes how the population of river sharks changes with time.

Trigonometric Models

Example:

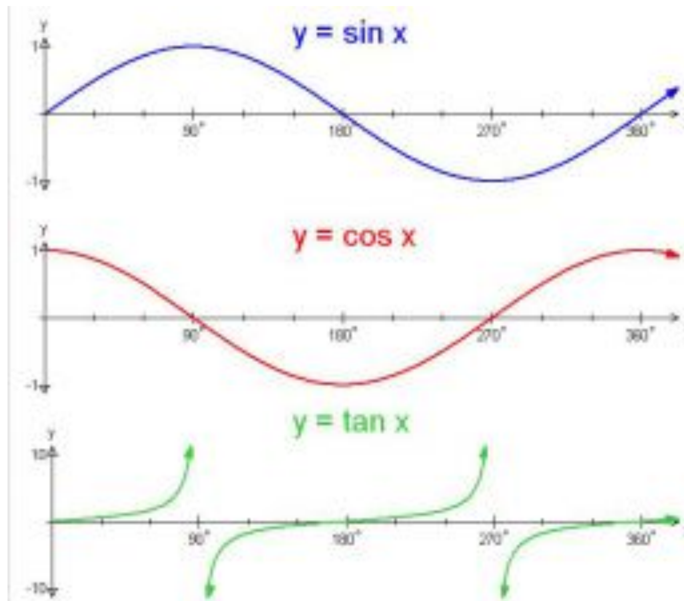
The spike trains in each input group were generated from the group's input intensity function. These are defined for each group of oscillatory inputs as

$$\begin{aligned} \hat{\lambda}_1(t) &= \hat{\nu}_0 + a \cdot \cos[2\pi f_m(t + \hat{d})], \\ \hat{\lambda}_2(t) &= \hat{\nu}_0 + a \cdot \cos[2\pi f_m(t + \hat{d} + \hat{d}_{lag})], \end{aligned} \tag{33}$$

where $\hat{\nu}_0$ is the mean input rate (in spikes/s), a is the amplitude in the oscillations (in spikes/s), f_m is the modulation frequency of the oscillations (in Hz), $\hat{d}$ is the delay of inputs in the first group (in seconds), and $\hat{d}_{lag}$ is the time lag between the oscillations of the two input groups (in seconds).

section 2.3

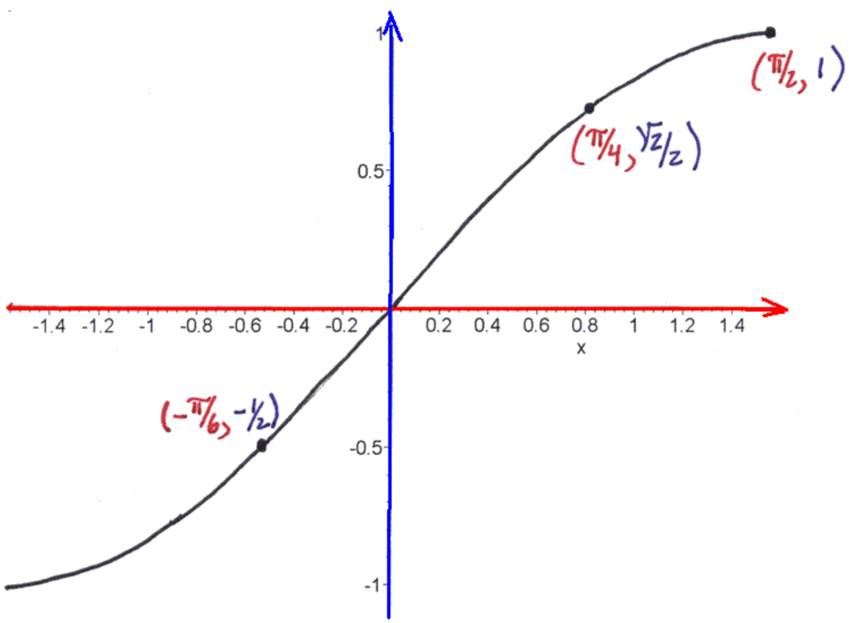
Inverse Trigonometric Functions



Since the 3 main trigonometric functions are *not* one-to-one on their natural domains we must first restrict their domains in order to define inverses.

Inverse of Sine

Restrict the domain of $f(x) = \sin x$ to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



Now the function is one-to-one on this interval so we can define an inverse.

Inverse of Sine

The inverse of the restricted sine function is denoted by $f^{-1}(x) = \sin^{-1} x$ or $f^{-1}(x) = \arcsin x$.

Cancellation equations:

$\arcsin(\sin x) = x$ $\quad \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (domain of sin x)

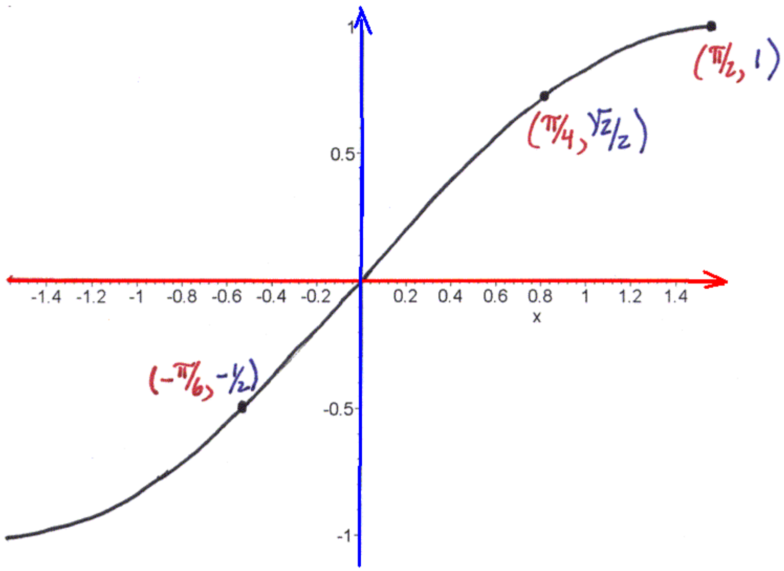
$\sin(\arcsin x) = x$ $\quad \forall x \in [-1, 1]$ (domain of arcsin x)

Calculate:

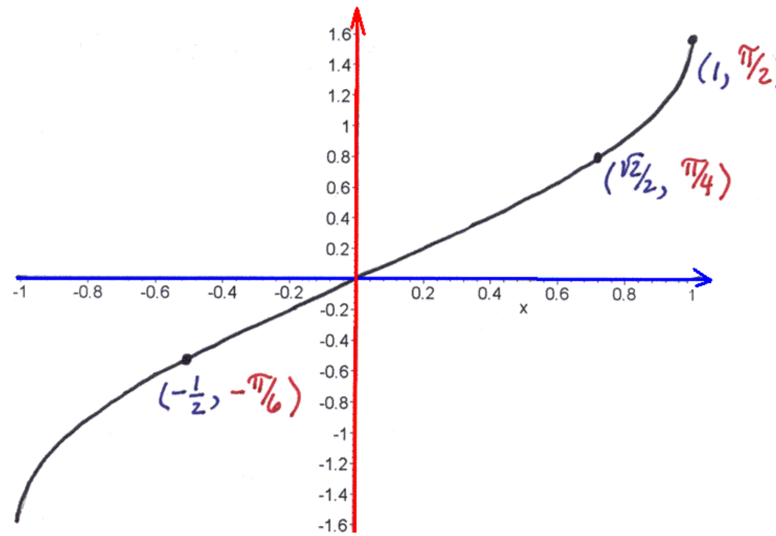
$\arcsin(\frac{1}{2})$ $\quad \sin(\arcsin(\frac{5}{7}))$ $\quad \arcsin(\sin(\pi))$

Graphs of Sine and Arcsine

y = sin x



y = arcsin x



domain: $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

range: $y \in [-1, 1]$

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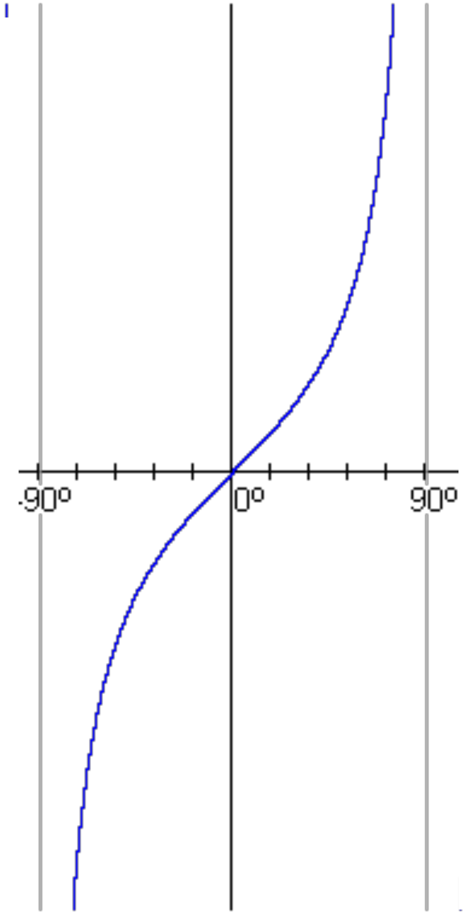
Inverse of Tangent

Restrict the domain of $f(x) = \tan x$ to $(-\frac{\pi}{2}, \frac{\pi}{2})$.



This portion of tangent passes the HLT so tangent is one-to-one here

Inverse of Tangent

The inverse of the restricted tangent function is denoted by $f^{-1}(x) = \tan^{-1} x$ or $f^{-1}(x) = \arctan x$.

Cancellation equations:

$\arctan(\tan x) = x \qquad \forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ (restricted domain of tan x)

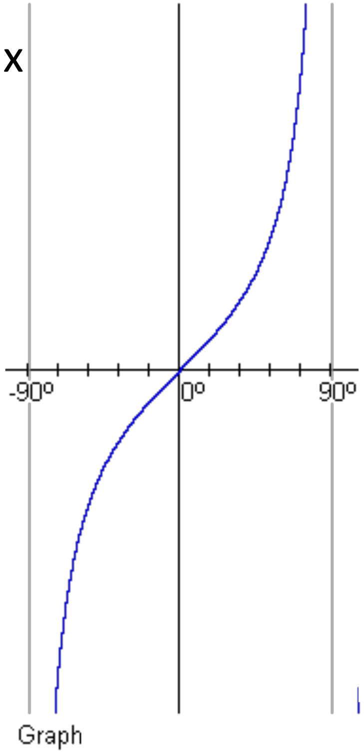
$\tan(\arctan x) = x \qquad \forall x \in (-\infty, \infty)$ (domain of arctan x)

Calculate:

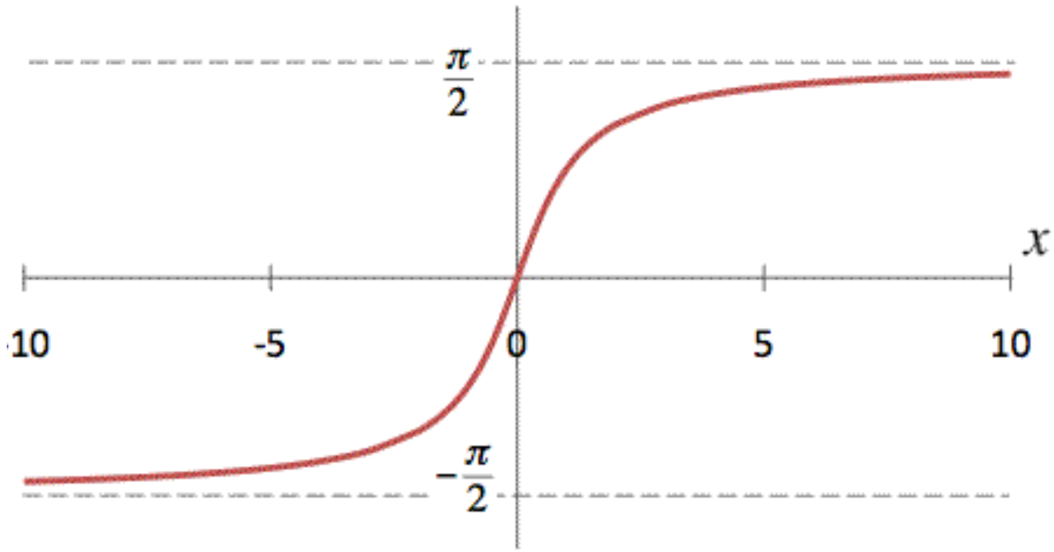
$\arctan(1) \qquad \tan(\arctan 10) \qquad \arctan(-\sqrt{3})$

Graphs of Tangent and Arctangent

y = tan x



y = arctan x



domain: $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

range: $y \in (-\infty, \infty)$

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range: $y \in (-\frac{\pi}{2}, \frac{\pi}{2})$

Real-life Use of Arctangent

Example: Model for World Population

One of the many models used to analyze human population growth is given by

$$P(t) = 4.42857\left(\frac{\pi}{2} - \arctan\frac{2007 - t}{42}\right)$$

where *t* represents a calendar year and *P*(*t*) is the population in billions.

What is the range of this function?