Dynamical Systems

- **Discrete-time dynamical systems describe a sequence of measurements made at equally spaced intervals**

- Continuous-time dynamical systems, usually known as differential equations, describe measurements that are collected continuously

Discrete-Time Dynamical Systems

A discrete-time dynamical system consists of an **initial value** and a **rule** that transforms the system from the present state to a state one step into the future.

Discrete-Time Dynamical Systems and Updating Functions

Let m represent the measurement of some quantity.

The relation between the initial measurement m_t and the final measurement m_{t+1} is given by the **discrete-time dynamical system**

$$m_{t+1} = f(m_t)$$

The updating function f accepts the initial value m_t as input and returns the final value m_{t+1} as output.

Note:
t represents present time and $t+1$ represents one time-step into the future

Solutions

Definition:

The sequence of values of m_t for t = 0, 1, 2, ...
is the **solution** of the discrete-time dynamical
system $m_{t+1} = f(m_t)$ starting from the **initial
condition** m_0.

The **graph** of a solution is a discrete set of points
with the time t on the horizontal axis and the
measurement m_t on the vertical axis.

Example:
A Discrete-Time Dynamical System for a Bacterial Population

Data:

Colony	Initial Population b_t (millions)	Final Population b_{t+1} (millions)
1	0.47	0.94
2	3.30	6.60
3	0.73	1.46
4	2.80	5.60
5	1.50	3.00
6	0.62	1.24

Example:
A Discrete-Time Dynamical System for a Tree Growth

Data:

Tree	Initial Height, h_t (m)	Final Height, h_{t+1} (m)
1	23.1	23.9
2	18.7	19.5
3	20.6	21.4
4	16.0	16.8
5	32.5	33.3
6	19.8	20.6

Example:
A Discrete-Time Dynamical System for Absorption of Pain Medication

A patient is on methadone, a medication used to relieve chronic, severe pain (for instance, after certain types of surgery). It is known that every day, the patient's body absorbs half of the methadone. In order to maintain an appropriate level of the drug, a new dosage containing 1 unit of methadone is administered at the end of each day.

Basic Solutions

<u>Basic Exponential Discrete-time Dynamical System</u>

If $b_{t+1} = rb_t$ with initial condition b_0, then $b_t = b_0 r^t$.

<u>Basic Additive Discrete-time Dynamical System</u>

If $h_{t+1} = h_t + a$ with initial condition h_0,

then $h_t = h_0 + at$.